Chapter 3

A model with continuous sectors

During the years, an odd trend may be observed in the economic geography models that were proposed. In the early papers, such as Fujita (1988) and Rivera-Batiz (1988), the space in which the location problem was solved was a one-dimensional line. The position on this line captures the distance from a central business district, and once the problem is solved in one dimension a two-dimensional solution is trivially available (This line of modelling goes back to von Thünen 1842). In later papers, starting from Krugman (1991a), the spatial structure was simplified to two possible locations with fixed transport costs between them. The internal ordering of the locations was left undiscussed. So, in some ways, the geographical predictions of new economic geography turned out to be weak.

One particular way of modelling linkages is used in Krugman and Venables (1995). In this paper, agglomeration occurs because of input-output connections: the production of one firm is used as an input for other firms, who therefore prefer to be close by. The authors assume that every firm uses as an input a composite good, made from the output of all other firms. In fact, the structure of production is completely symmetric, so that each firm uses every possible product to the same degree. This is a natural property of the Dixit-Stiglitz framework, in the way it is usually applied.

It is not impossible to step away from the assumed symmetry and allow for some variety in the input structure. Young (1993) introduces a production function with a continuum of intermediate goods suppliers and a continuum of final goods suppliers. The latter use intermediate goods, but only if these goods come from producers that are sufficiently 'close' to their product, where closeness is defined within the continuum of firms.

In Section 3.1, I introduce a similar adaptation of the Dixit-Stiglitz framework. On top of this adaptation, we can construct a model of economic geography à la Krugman and Venables (1995). For a few simple input-output patterns, this model shows that a greater variety of equilibria may be obtained than just agglomeration or symmetry. We develop and experiment wit this model in Section 3.2. We look at a slightly more complicated form of the input-requirement function, which defines our sectors, in Section 3.3. In this section, we prove the existence of an equilibrium in which 'old' firms cluster in one region, while 'new' firms agglommerate in the other. This result allows us to characterize the development of different regions in a growing economy. Section 3.4 concludes.

3.1 Generalized Monopolistic Competition

We look at production that takes place in firms that operate in a monopolistic competitive[1] market. As inputs, these firms use labor and composite of output of other firms. The production function is of the Cobb-Douglas variety,

$$z_i = L_i^\alpha Q_i^{1-\alpha}. \tag{3.1}$$

The set of firms is assumed to be a continuum $[0, n]$ [2] Each firm produces a single product that may likewise be indexed on $[0, n]$. We can now define the composite Q_i, that firm i uses as an input, as a CES-aggregate of those products.

$$Q_i = \left[\int_{f(i)} \left(x_i^j \right)^\theta \mathrm{d}j \right]^{1/\theta} \tag{3.2}$$

where x_i^j is the amount of input j used by firm i and $0 < \theta < 1$. Here, we use the function $f(i) : [0, n] \to S_n$, where S_n is the sigma field of all open and closed intervals on $[0, n]$. This function indicates what array of inputs firm i uses, and we require nothing of it except that it does not map to the empty set so that each firm uses at least a positive measure of intermediate inputs.

In the usual application, $f(i)$ is the entire set $[0, n]$ for all i. This accounts for the symmetry in production structure that we discussed in the introduction. Young (1993) uses the function $f(i) = [Bi, \min(\Theta i, n)]$ where $0 \leq B < 1$ and $\Theta > 1$.

Because of the continuum of firms that we assumed, this modification of Dixit and Stiglitz (1977) does not alter the market structure very much. Firms still face an MC market where their pricing decision does not affect the general price level. More specifically, a producer i who spends an

[1] We will use the acronym MC from now on.

[2] For an interpretation of such a continuum of firms, see section 2.A

amount E on intermediate goods will demand of good j the amount x_i^j:

$$\begin{aligned} x_i^j &= \frac{Ep(j)^{-\sigma}}{\int\limits_{f(i)} p(k)^{1-\sigma} \, \mathrm{d}k} \text{ if } j \in f(i) \\ &= 0 \text{ if } j \notin f(i). \end{aligned}$$

where $\sigma = 1/(1-\theta) > 1$. This shows that producers are confronted with a constant price-elasticity of demand σ, when the demand is for intermediate goods. If final demand for the goods also has the same elasticity σ, the optimal pricing decision is to set prices as a markup over marginal costs. The optimal markup is $\sigma/(\sigma-1)$.

The price index for producer j's intermediate good is

$$p_Q^j = \left[\int\limits_{f(j)} p(k)^{1-\sigma} \, \mathrm{d}k \right]^{\frac{1}{1-\sigma}}. \tag{3.3}$$

This is the 'ideal' price index (see Green 1964) so that $p_Q^j Q_j = E$.

Based on the intermediate-input function $f(i)$ we can define an 'inverse' function $g(i)$ that maps the index of a producer, i, into the set of intermediate producers that use her good:

$$\begin{aligned} g(i) &: [0,n] \to S_n \\ g(i) &= \{j \in [0,n] | i \in f(j)\}. \end{aligned}$$

We assume that all intermediate goods are used somewhere, so that the function $g(j)$ does not map to the empty set for any j in $[0,n]$. Using this function, we can write the demand for a specific intermediate good j as

$$\begin{aligned} x^j &= \int\limits_{g(j)} \frac{E(i)p(j)^{-\sigma}}{\int\limits_{f(i)} p(k)^{1-\sigma} \, \mathrm{d}k} \mathrm{d}i \\ &= p(j)^{-\sigma} \int\limits_{g(j)} \frac{E(i)}{\int\limits_{f(i)} p(k)^{1-\sigma} \, \mathrm{d}k} \mathrm{d}i \end{aligned}$$

Notice that producer j still faces a demand curve with constant elasticity σ, as in the MC setup. The price that maximizes profit will therefore be a markup $\sigma/(\sigma-1)$ times marginal cost, as usual.

3.1.1 An example

To gain some insight into the effects of this modification, let us look at two examples that will prove useful later on. We first model a recursive[3] MC

[3] The term 'recursive' is used to indicate that the group of producers uses some of its own product as input (see eqn. 3.1)

economy where there is no variety in input structure, and then one where there are two distinct sectors. The latter example can be generalized to N distinct sectors.

Example 1: A one-sector economy

We consider an economy with L workers that supply one unit of labor inelastically. We use the wage rate as numéraire and set it to one. There is a continuum $[0, n]$ of firms and n is fixed. Producers face production functions (3.1) and (3.2). In this example, we take $f(i) = [0, n]$ for all i. This implies that a firm's cost function is

$$\begin{aligned} C\left(p_Q, z_i\right) &= (1-\alpha)^{\alpha-1}\,\alpha^{-\alpha} w^{\alpha} p_Q^{1-\alpha} z_i \\ &= (1-\alpha)^{\alpha-1}\,\alpha^{-\alpha} p_Q^{1-\alpha} z_i \end{aligned} \tag{3.4}$$

The price index p_Q is defined in (3.3); the superscript i is omitted because it is the same for all firms, due to our assumption about $f(i)$.

Consumers maximize utility U, given by

$$U = \int_0^n \left(x_i^j\right)^{\theta} \mathrm{d}j. \tag{3.5}$$

Notice the crucial assumption that the parameter θ is the same in this utility function and in production function (3.2). A producer, whose product is demanded from consumers as well as other producers, now faces two demand curves with the same constant elasticity σ. This reduces the problem, as the optimal price is simply a markup $\sigma/(\sigma-1)$ times the marginal cost, or

$$p = \frac{\sigma}{\sigma-1}\,(1-\alpha)^{\alpha-1}\,\alpha^{-\alpha} p_Q^{1-\alpha}$$

from (3.4). Because p_Q is again a function of p as in (3.3), we can simplify to

$$p = \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{\alpha}} (1-\alpha)^{\frac{\alpha-1}{\alpha}}\,\alpha^{-1} n^{\frac{1-\alpha}{\alpha(1-\sigma)}} \tag{3.6}$$

where we have used the fact that all prices are equal in equilibrium. Notice that this formula completely fixes the price in terms of parameters. We can now also solve for the other endogenous variables.

The price index for the composite good Q is, by (3.3), equal to

$$p_Q = p n^{\frac{1}{1-\sigma}}.$$

Because labor supply is exhausted, we know that each 'unit of firms' applies L/n units of labor in equilibrium, which is also the sum it pays out in wages. From the Cobb-Douglas structure of (3.1), we know that this must be a fraction α of total costs. Therefore,

$$Q_i = \frac{L}{n}\frac{1-\alpha}{\alpha}\frac{1}{p_Q} \tag{3.7}$$

for all i. This is the amount of the composite good that firm i demands. From each specific producer j, an amount x_i^j is demanded, where

$$\begin{aligned} x_i^j &= Q_i p(j)^{-\sigma}\left[\int_0^n p(k)^{1-\sigma}\mathrm{d}k\right]^{\frac{\sigma}{1-\sigma}} \\ &= Q_i n^{\frac{\sigma}{1-\sigma}}, \end{aligned}$$

where we used the fact that all prices are equal in equilibrium. This means that each firm j faces an intermediate demand for its good equal to

$$\int_0^n x_i^j \mathrm{d}i = Q_i n^{\frac{1}{1-\sigma}}. \tag{3.8}$$

We now look at final demand. Because firms use a constant returns technology and price as monopolists, they will earn a nonzero profit. We assume that each inhabitant owns an equal stake in each firm, so that these profits are equally divided among them. This puts per capita income at

$$y = 1 + \pi\frac{n}{L} \tag{3.9}$$

where π is the profit of a firm. From the markup pricing rule, we know that profits are a fraction $1/\sigma$ of wholesale, so that

$$\pi = \frac{1}{\sigma}z_j p. \tag{3.10}$$

The total expenditure on all goods then is Ly, so allocated that (3.5) is maximized. This gives a per-firm final demand of

$$y_j = L\frac{y}{p_Q}n^{\frac{\sigma}{1-\sigma}} \tag{3.11}$$

As a check on our computations, it is now possible to show that final plus intermediate demand per firm is equal to a firm's production, or

$$L\frac{y}{p_Q}n^{\frac{\sigma}{1-\sigma}} + Q_i n^{\frac{1}{1-\sigma}} = \left(\frac{L}{n}\right)^{\alpha} Q^{1-\alpha}. \tag{3.12}$$

This is done in Appendix 3.A.

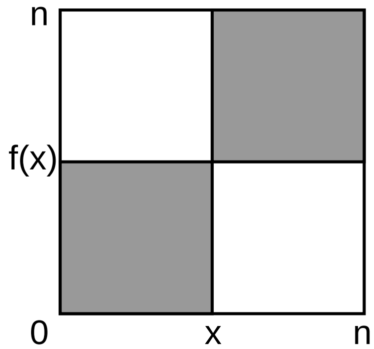

Figure 3.1: Function $f(x)$ in formula (3.13) mapped out.

Example 2: Two distinct sectors

We use the same setup as in the previous paragraph, with the exception that we alter $f(x)$ and use

$$f(x) = \begin{cases} [0, \frac{1}{2}n) & \text{if} \quad x \in [0, \frac{1}{2}n) \\ [\frac{1}{2}n, n] & \text{if} \quad x \in [\frac{1}{2}n, n] \end{cases} \tag{3.13}$$

With this f, there are two sectors in the economy, separated at $x = n/2$ in the continuum. Firms only use intermediates from their own sector, and consequently only receive demand for intermediates from their own sector. In this specific case, we have $f(x) = g(x)$ for all x.

It is possible to visualize $f(x)$, as is done in Figure 3.1. The index x is on the horizontal axis, while $f(x)$ may be read of the vertical axis as the grey area above x. Alternatively, the function $g(x)$ could be read of the horizontal axis with x on the vertical. In this case, both functions are the same.

We solve the problem along the same lines as in section 26. As a consequence of the change we made, we must study some of the characteristics of the two sectors separately. However, because the sectors are exactly the same, we can suffice with the specification of one of them; the same results will hold for the other.

Within the first sector, $[0, n/2)$, the price index for the composite good is different from above, and now reads

$$p_Q^j = \left[\int_0^{\frac{1}{2}n} p(k)^{1-\sigma} \mathrm{d}k \right]^{\frac{1}{1-\sigma}}. \tag{3.14}$$

Therefore, the price in sector 1 will be

$$p = \left(\frac{\sigma}{\sigma - 1} \right)^{\frac{1}{\alpha}} (1 - \alpha)^{\frac{\alpha - 1}{\alpha}} \alpha^{-1} \left(\frac{1}{2} n \right)^{\frac{1-\alpha}{\alpha(1-\sigma)}} \tag{3.15}$$

Note that this is formula (3.6) multiplied by $(1/2)^{(1-\alpha)/(\alpha[1-\sigma])}$. Because the last exponent of this expression is negative, we see that this price in (3.15) is higher than the price in (3.6). This reflects the restriction that we have imposed on sector 1 firms: they can only use inputs from a subset of firms. Because there are increasing returns to the scale of available inputs (see chapter 2), a decrease in scale will increase the costs and therefore the price.

From (3.14) we have that $p_Q^j = p(\frac{1}{2}n)^{\frac{1}{1-\sigma}}$, which is used in the unchanged formula for Q_j, formula (3.7). Knowing Q_j, we know how much each firm in the sector spends on the intermediate composite good. For every single firm, this means a demand of $Q_j(\frac{1}{2}n)^{\sigma/(1-\sigma)}$ from every firm in its sector, leading to a per-firm demand for intermediate purposes of

$$Q_j(\frac{1}{2}n)^{\sigma/(1-\sigma)} \cdot \frac{1}{2}n = \frac{1}{2}L\frac{1-\alpha}{\alpha}\frac{1}{p_Q^j}\left(\frac{1}{2}n\right)^{\frac{\sigma}{1-\sigma}}$$

As for final demand, the public still consumes *all* goods and maximizes (3.5). Therefore, the price index faced by the public still is

$$p_Q = pn^{\frac{1}{1-\sigma}} \tag{3.16}$$

with p from (3.15). This index is used in (3.11), while formulae (3.9) and (3.10) still hold in this model.

Example 3: N distinct sectors

We can generalize the above case further by taking the number of sectors a variable N and using the function

$$f(x) = \begin{cases} [0, \frac{1}{N}n) & \text{if} \quad x \in [0, \frac{1}{N}n) \\ [\frac{1}{N}n, \frac{2}{N}n) & \text{if} \quad x \in [\frac{1}{N}n, \frac{2}{N}n) \\ \vdots & \vdots \qquad \vdots \\ [\frac{N-1}{N}n, n] & \text{if} \quad x \in [\frac{N-1}{N}n, n] \end{cases} \tag{3.17}$$

A map of the function is in Figure 3.2 for $N = 5$. Again we assume that firms in a sector only use products from their own sector.

Instead of solving the entire model again, this time we make use of an interesting regularity in the outcomes. From the inelastic supply of labor, the fact that wage is fixed at 1 and the number of firms is fixed, we know that the wage bill per firm is always the same, L/n. Due to the Cobb-Douglas structure of function (3.1) this also fixes the amount of money spent on intermediates. From that amount, it is possible to compute that a constant fraction ϕ of the production of any firm is used as intermediates by other firms in the same sector. there holds that

$$\phi = (1-\alpha)\,\frac{\sigma-1}{\sigma},$$

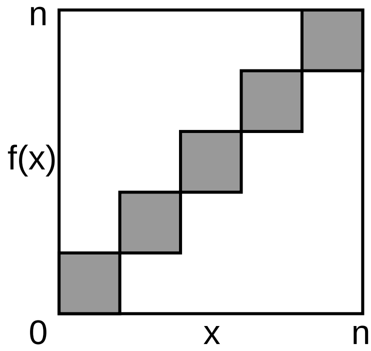

Figure 3.2: Function $f(x)$ in formula (3.17) mapped out.

so that ϕ independent of N.

The number ϕ can help compute output quickly. This works as follows: we know that the size of a sector is n/N, which determines the intermediate market, and is also the length over which the integral and p_Q^j is computed. Knowing this size, we can compute p as in (3.6) and (3.15). From p and the size, we compute p_Q^j, and from this we know Q from (3.7), which in turn fixes intermediate demand. And with that in hand, we can use ϕ to determine total production per firm.

It turns out that production per firm is

$$z_j = \phi^{\frac{1-\alpha}{\alpha}} \frac{L}{n} \left(\frac{N}{n} \right)^{\frac{1-\alpha}{\alpha(\sigma-1)}} .$$

In this expression we can clearly see the effect of dividing up the economy into different sectors. The exponent on N is negative, so that the more sectors there are, the less is produced in total. This is caused by the fact that there are increasing returns to the number of varieties available (or, usable) as intermediate input.

It is important to bear in mind that the above result hinges on the fact that there is no inter-sector trade. This assumption is not realistic, but can easily be relaxed.

3.2 A two-sector model of economic geography

In this section, we will use the model of example 2, section 3.1.1, in an environment with two locations and nonzero transport costs. This setup is reminiscent of Krugman and Venables (1995), albeit that the economy now has two, separate, sectors.

3.2.1 Sectors that only use their own product

Imagine there are two possible locations, N and S. Transport costs are of the iceberg-kind as in Samuelson (1952): only a fraction τ of the goods that are shipped actually arrives. Each location has population $L/2$, which is the amount of labor supplied with elasticity zero. The wage rate in N is one by normalization, that in S is w.

We again assume a continuum of firms with length n. Of these firms, those in $[0, n/2)$ are in sector 1 and those in $[n/2, n]$ are in sector 2. The measure of firms per sector is invariant. We denote by $m_1 \in [0, n/2]$ the measure of firms from sector 1 that reside in location N. That leaves $n/2 - m_1$ firms from sector 1 in location S. Similarly, $m_2 \in [0, n/2]$ firms from sector 2 reside in location N.

Firms use labor and an intermediate composite good, which comprises output from all the firms in their own sector. The price index for the composite good in section 3.1.1 was given by (3.14). Now that there are transport costs, this index is a little more complicated. For a firm in sector j and location λ, it is

$$p_Q^{j,\lambda} = \left[\int_0^{n/2} \left(\frac{p(k)}{\tau^{|\lambda - L(k)|}} \right)^{1-\sigma} \mathrm{d}k \right]^{\frac{1}{1-\sigma}} \tag{3.18}$$

where $j \in \{1, 2\}$ is the sector and L indicates the location of firm k:

$$L(k) = \begin{cases} 0 & \text{if firm } k \text{ is in } N \\ 1 & \text{if firm } k \text{ is in } S \end{cases} .$$

The same convention holds for the variable λ.

As above, we assume that final demand is for a composite of all goods from both sectors. The price index for that composite, previously given by (3.16), now is different for the two locations, and equal to (3.18) with the integral along $[0, n]$.

Producers face a demand curve that is an aggregate of demand from firms in two regions and consumers in two regions. Because the costs of transport are just a multiple of wholesale, and because consumers and firms share parameter θ, this curve has a constant elasticity of demand σ. Optimal prices are therefore a markup $\sigma/(\sigma - 1)$ over marginal cost.

The solution to this model may now be derived. All price indices take the form

$$\left(\sum_{i=1}^{l} \Phi_i \right)^{1/(1-\sigma)}$$

with l an integer larger than one. Contrary to section 3.1, the terms Φ_i now vary with i. This form cannot be simplified, so that we must rely on computational methods to approximate a solution. This necessity almost

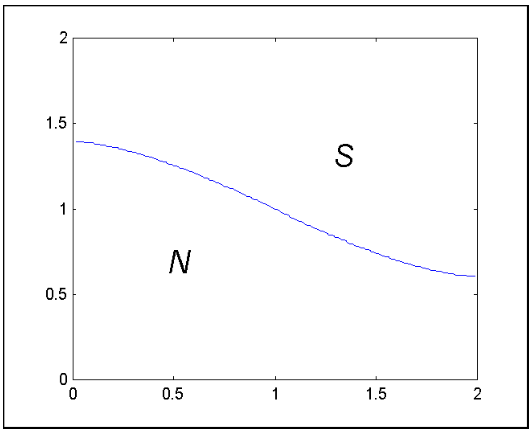

Figure 3.3: Sector 1 preferable region

always arises in new-geography models, and is recognized by Krugman (1998).

The approximation works as follows. Given the wage, prices can be computed. Given the amount of labor hired by each firm (which must integrate to $L/2$ for each location), production and demand can be computed.

With those results, we can look at how the parameters that were taken as given should be modified. Labor hired reacts to excess demand or supply of goods in a sector-location couple. Wage responds to excess demand or supply between the two locations.[4] The model converges until all demand, intermediate and final, is equal to supply.

With the above solution, we have taken the measure of firms per location, m_1 and m_2, as given. One of the results from the model is that we can compute the profit per firm, as a function of location and sector. This is because all firms in the same sector and in the same location behave alike, and have the same profit. Looking at the pattern of profits can give some insight into the possible migration patterns of firms, assuming that they are driven by profit maximization. Note that a migrating firm leaves its laborers behind and hires from the other pool, so that the number of inhabitants remains equal between the locations.

The results are in the two figures above. The variable m_1 is on the horizontal axis, m_2 is on the vertical axis. The left panel shows the preferred location for firms in sector 1, given m_1 and m_2. The preferred location is the location where the profits per firm are higher. The same diagram is drawn in the right panel, for sector 2. The other parameters in this model were $n = 4$, $L = 40$, $\sigma = 3$, $\tau = 0.8$ and $\alpha = 0.6$.

To find the agglomeration pattern that might result if the firms actually

[4]Instead of varying the wage we could also have specified an exchange rate between the two locations and set both wage rates to one—in their own currency.

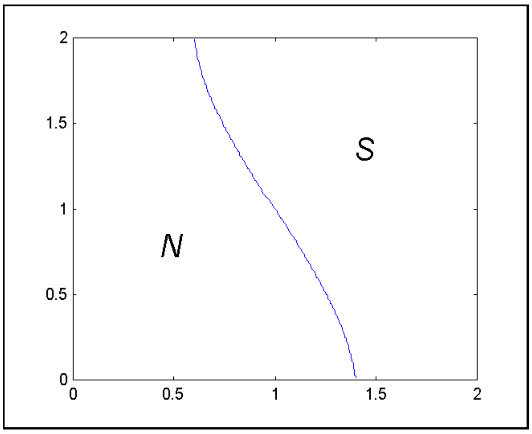

Figure 3.4: Sector 2 preferable region

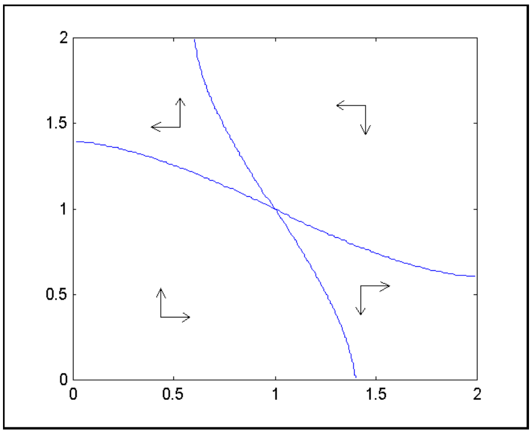

Figure 3.5: Direction of motion in the (m_1, m_2) plane. m_1 on the horizontal axis, m_2 vertical.

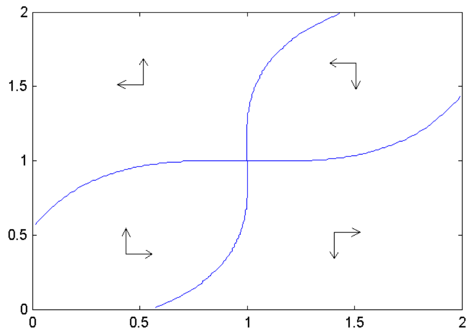

Figure 3.6: Dynamics when $\tau = 0.2$

responded to the incentives given by the profit rate, we combine the two panels in figure 3.5. This figure shows that the model, with the current parameters, tends to correct imbalances. If there are few firms of both sectors in region N (low m_1, m_2), there will be migration toward that region. However, if the imbalance is such that there are a lot of firms of one sector in the north, while most of the firms of the other sector are in the south, the tendency is toward complete separation of the two sectors. There are three long-term equilibria in this model: the saddle-point stable equilibrium $(m_1, m_2) = (n/4, n/4)$ and the stable equilibria $(0, n/2)$ and $(n/2, 0)$.

The precise long-term result depends on how the laws of motion of the firms are specified, and on the initial condition. If region N historically has a lot of sector 1 activity, while region S is the historic center for sector 2, we see that the model reinforces that structure.

This result is interesting because it is reminiscent of many other results in economic geography. By that I mean the dependence on initial conditions and complete agglomeration of sectors. However, the division of the economy into sectors adds to the credibility of the outcome. No longer does *all* activity agglomerate into one location, as previous results showed, but we have a situation where the agglomeration is per sector. This is because the economies of scale that drive agglomeration are present within a sector, but the diseconomies of scale (*e.g.*, rising wages) are present between sectors.

It can be interesting to modify the parameters a bit and to check the effects on the outcome. In figure 3.6, we increased transport costs tremendously by setting $\tau = 0.2$. It turns out that there still are three equilibria with the same stability properties. This is typical for all values of $\tau < 1$.

3.2.2 Other IO patterns

We look at a number of other patterns of input-output between sectors, using the same methodology as above. First, consider the two-sector situation

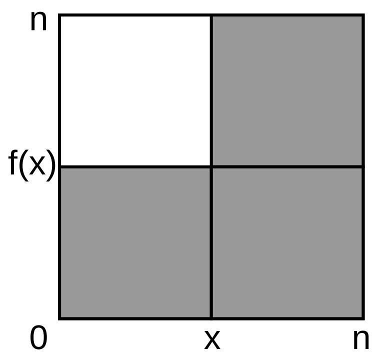

Figure 3.7: The function $f(x)$ in formula (3.19) mapped out

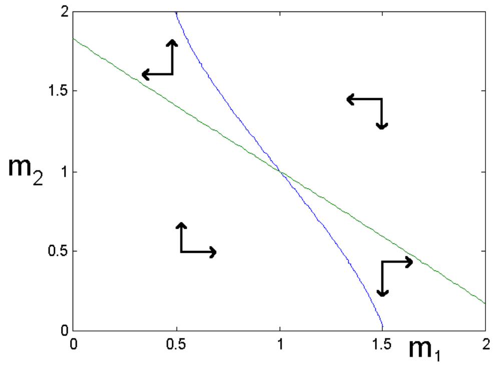

Figure 3.8: Direction of motion when sector 1 uses all output as intermediate, sector 2 uses only sector 2 output.

where sector one uses a composite of *all* output as intermediate and sector two uses only its own output. The function f in this case is

$$f(x) = \begin{cases} [0, n] & \text{if } 0 \leq x \leq n/2 \\ [n/2, n] & \text{if } n/2 < x \leq n \end{cases} \tag{3.19}$$

In this case, if there is a region with many sector two firms, the firms from sector one face conflicting incentives. Because of the 'crowding out' phenomenon, wages will be relatively high in that region, so that it is relatively unattractive. On the other hand, the new input-output structure shows that there is an advantage in being close to sector two firms, as that is where sector one firms get some of their inputs from. The contradictory incentives are clear from figure 3.8, with the directions of motion.

We see that for high values of m_2, and low values of m_1, there is almost always an incentive to increase m_1. Compared with figure 3.6, the two indifference curves have moved towards each other, leaving a smaller

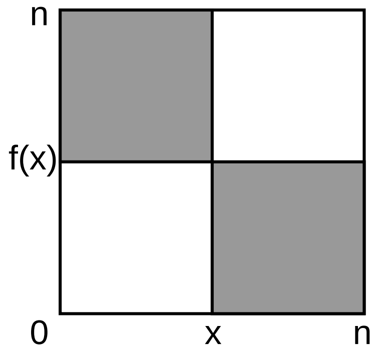

Figure 3.9: The function $f(x)$ in formula (3.20) mapped out.

area with divergent behavior. Figure 3.8 uses the same parameters as the simulation in figure 3.6 above.

It is not surprising that for sector two firms, the crowding-out effect is also less. The presence of a large number of sector one firms increases the demand for their product, which makes the location more attractive. However, we see that of the three equilibria $(2,0)$, $(0,2)$ and $(1,1)$, still only the first two are stable. So, we still have a model where the long-term solution is that each sector finds its own region.

We can also do a complete reversal of the model in Section 3.2.1 and adapt the input-output function

$$f(x) = \begin{cases} [n/2, n] & \text{if } 0 \leq x \leq n/2 \\ [0, n/2] & \text{if } n/2 < x \leq n \end{cases} \tag{3.20}$$

This function specifies that firms in sector one use a composite of sector two output as input, and vice versa. The sectors do not use any of their own output as intermediate.

The results are in Figure 3.10. In this case, there is no advantage for firms in being isolated with their own kind. There is only one equilibrium, and it is the stable equilibrium $(1,1)$.

3.3 Location, sectors and economic growth

3.3.1 Introduction

In the previous paragraph, we have shown how it can be in the interest of industries from different sectors to settle in different regions. The strength of the ties within and between those sectors, which is measured by the input-output function, determines the equilibrium location outcome. Which industry settles in one particular region is indeterminate, but once

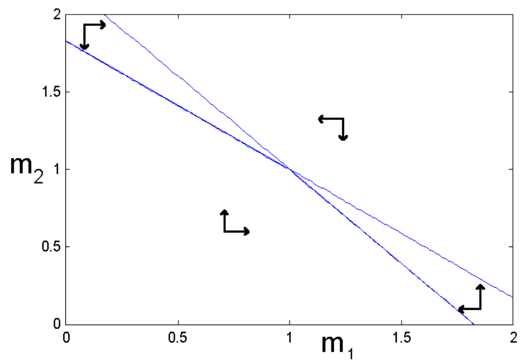

Figure 3.10: The direction of motion when sectors use each others output as intermediate input.

the equilibrium is attained it may self-perpetuate. If the ties between sectors are such that firms like to settle close to other members of the same sector, as in section 3.2.1, the initial characteristics of a region are locked in when new firms arrive. The new firms will locate close to members of their own sector.

This outcome would allow us to make certain observations on the development of regions, if we are prepared to assume something about the ties that new firms will have with different existing sectors in the economy. We will make such an assumption in this section. Specifically, we will assume that ties between firms are stronger if they are approximately from the same period. We will propose a specific form of the IO-function $f(x)$ that reflects this assumption and use it to derive a spatial equilibrium between two regions. The specific form of $f(x)$ will then allow us to discuss the evolution of this equilibrium in the context of a growing economy.

Throughout this chapter, we have assumed that the number of firms n is fixed, indicating that there is no free entry for firms. We now expound on this assumption: in our model, specific technical knowledge is required to start a firm. This knowledge is proprietary and can only be used to start a single firm. The assumption allows the owners of the firm to turn a profit, something that is impossible under free entry. Up to now, we have kept looked at a situation in which the number of firms is constant; in this situation, no new technical developments take place.

The economy as we observe it, with a continuum of n active firms, is the result of technological progress from the past. We will assume the following about the nature of this progression: each new production process is partly the result of insights gleaned from a number of recent previous innovations, and uses the products of those innovations as inputs. In turn, a new production process can itself inspire new products. Those new products can then serve as an input into the original process, making it more

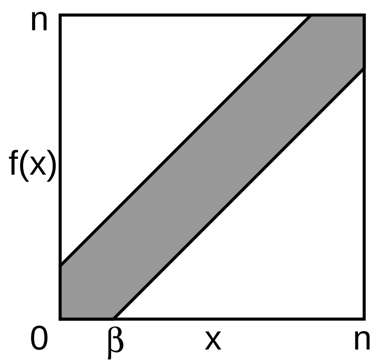

Figure 3.11: Function $f(x)$, which shows which intermediate goods each firm $[0 \ldots n]$ uses, as defined in formula (3.21).

efficient. In this setup, each firm uses as intermediate inputs those products that are 'close' to its own production process in a technical sense. This assumption was used by Young (1993), for instance, to explain certain aspects of economic growth.

In a sense, our previous example 2 on page 60 is a crude representation of this assumption. Firms in one sector can be thought to be technologically close to each other. If we view the order of firms on the line $[0 \ldots n]$ as the result of a technological progression, a new technological paradigm was introduced in firm $n/2$ which resulted in incompatibility between the two sectors.

We will abstain from such shifts in paradigm in our example and use a continuous mapping $f(x)$, defining a gradual scale of technological processes. We use

$$f(x) = [\max(x - \beta, 0), \min(x + \beta, n)] \tag{3.21}$$

where $\beta > 0$. Again, $f(x)$ is the range of firms from which firm x uses the output as an intermediate input. A graphical representation of this function is given in figure 3.11. The parameter β is a measure of how 'close' firms have to be to another firm, in order use their product as an intermediate input. For large values of this parameter, if $\beta > n$, we are back at the first model of this chapter where each firm uses every available product as an intermediate input. For smaller β's, firms only use the products of their technological neighbors. Note that for firms close to n, the set of available intermediate goods is smaller than for those in the middle. This is borne out by the minimum-condition in formula (3.21). These firms would like to make use of more advanced intermediate products, made by firms with an index larger than n. These, more advanced, products have not yet been developed.

We will discuss a particular spatial equilibrium for an economy in which this function $f(x)$ determines the input-output structure. In our discussion, we will leave the value of β unspecified. This will allow us to make a few comments about regional specialization and economic growth afterwards. Economic growth, driven by technological innovations, can be seen in this model as an increase in n, the number of firms. However, an increase in n is isomorph with a *decrease* in β. Because our equilibrium holds for all values of β, it is not affected by economic growth. We will return to this subject after our discussion of the spatial equilibrium.

3.3.2 The spatial equilibrium

We use the model of the previous sections, including the spatial setup from paragraph 3.2: there are two regions with transport costs between them, so that only a fraction $\tau < 1$ of transported goods arrives. For a function that determines the nature of the intermediate goods aggregate for each firm x, we use the specification in (3.21).

In this paragraph, we will prove that the symmetric equilibrium of this model is stable. The symmetric equilibrium is given by the following solution to the location problem: firms with an index between 0 and $n/2$ locate in one region, firms with an index between $n/2$ and n locate in the other. Throughout the section, we will assume that firms with a lower index locate in the N region. This assumption is, of course, immaterial.

We prove that the symmetric equilibrium is stable by showing that, given that the equilibrium has obtained, no firm would want to deviate from it. This does not prove that other types of equilibria are impossible; we simply show the existence of this particular equilibrium. The fact that the solution is symmetric simplifies the proof considerably: apart from the numbering of the firms, the situation of firm $i < n/2$ in region N is exactly identical to that of firm $n - i$ in region S. One result of this symmetry is that wages in both regions are the same. We will exploit the symmetry of the equilibrium throughout the proof.

Consider firm i, with $i < n/2$, in region N. We will show that this firm does not want to move from its home region to region S. Because of the requirements to start a firm (specific proprietary technical knowledge) it is impossible to copy a version of this firm to the other region, so that the decision not to move is leaves the equilibrium in place. As stated above, the economic circumstances of firm i are identical to those of firm $n - i$ in the other region. By proving that firm i does not want to leave, we will have proved the same about firm $n - i$, and thus about all the firms in region S.

The profits of firm i are determined by a number of factors, some of which vary with the region in which the firm operates. These factors are: local wage, final demand, intermediate demand and intermediate costs. Symmetry implies that local wage is the same in the two regions, as is final demand: we retain the assumption that the (identical) consumers of both regions want each available product equally much. The location decision of firm i, which is infinitely small, does not change this. Thus, we only look at intermediate demand and the costs of intermediate goods for firm i in each region. We will show that the costs are lower in region N, and that demand is higher in that region.

First we look at intermediate costs. Consider two possible cases: in the first case, $i + \beta \leq n/2$. This means that all intermediate goods suppliers of firm i are located in the same region, the home region N. It is immediately obvious that relocating to the other region will increase intermediate costs with the transport markup. In the second case, $i + \beta > n/2$. We look at the price index of intermediate goods for firm i, which is similar to formula (3.18). If we use the the same definition of the location function $L(k)$ as on page 63, it is is equal to

$$\begin{aligned}\left(p_Q^{i,N}\right)^{1-\sigma} &= \int_{\max(i-\beta,0)}^{\min(i+\beta,n)} \left(\frac{p(k)}{\tau^{L(k)}}\right)^{1-\sigma} \mathrm{d}k \\ &= \int_{\max(i-\beta,0)}^{\frac{n}{2}} p(k)^{1-\sigma}\mathrm{d}k + \int_{\frac{n}{2}}^{\min(i+\beta,n)} \left(\frac{p(k)}{\tau}\right)^{1-\sigma} \mathrm{d}k\end{aligned}$$

The second step follows from the definition of the equilibrium, which stipulates that firms $[0 \dots n/2]$ are in region N. The second integral is taken over a positive domain in this case, indicating that some intermediate goods suppliers are in the other region. Moving to this region will decrease the costs of their products, but increase those of the region-N suppliers. We rewrite the expression to find out about the balance between the two:

$$\left(p_Q^{i,N}\right)^{1-\sigma} = \int_{\max(i-\beta,0)}^{\frac{n}{2}} \left[p(k)^{1-\sigma} + \left(\frac{p(k)}{\tau}\right)^{1-\sigma}\right] \mathrm{d}k + \int_{\frac{n}{2}}^{n-i-\beta} p(k)^{1-\sigma}\mathrm{d}k$$

where we have used the fact that, due to symmetry, $p(n-k) = p(k)$. Using similar steps, we can write the costs of intermediate goods for the same firm, if it relocates to the S-region, as

$$\left(p_Q^{i,S}\right)^{1-\sigma} = \int_{\max(i-\beta,0)}^{\frac{n}{2}} \left[p(k)^{1-\sigma} + \left(\frac{p(k)}{\tau}\right)^{1-\sigma}\right] \mathrm{d}k + \int_{\frac{n}{2}}^{n-i-\beta} \left(\frac{p(k)}{\tau}\right)^{1-\sigma} \mathrm{d}k$$

Because of the nonnegative domain of the second integral, the fact that prices are positive and because $\tau < 1$, we see that $p_Q^{i,N} < p_Q^{i,S}$. That is, intermediate goods for firm i are cheaper in the N region than in the S region.

We now turn to the intermediate demand for firm i. As a result of the IO-function $f(x)$ in (3.21), firm i receives intermediate demand from firms between $\max(0, i-\beta)$ and $\min(n, i+\beta)$. We again divide these firms in two groups, those in the home (N) region and those in the foreign region. If all demanding firms are in the home region (when $i + \beta \leq n/2$), then moving to the foreign region will certainly decrease intermediate demand because of transport costs. If not, we can identify a subset of the demanding firms, the group $[n - i - \beta, i + \beta]$. Total intermediate demand from this group for firm i will be exactly the same, regardless of where firm i decides to locate. This follows from the symmetry of the initial equilibrium and the symmetry of this particular group around $n/2$. The rest of the firms that demand intermediate goods from firm i, $[\max(0, i - \beta), n - i - \beta]$ is located in the home region N. This means that their demand will be higher if firm i is also located in this region. Taking the separate conclusions together, we see that intermediate demand for firm i is higher if it decides to locate in the initial region N.

Firm i resides in region N, according to the initial equilibrium. We have shown that moving to the other region would change two things for this firm: intermediate goods become more expensive and intermediate demand drops. This will make a move to the other region unattractive: both effects will serve to decrease its profit. We conclude that the position of the firm in the initial equilibrium is stable. This proofs that the equilibrium itself is stable, as the arguments holds for all firms in N and, by symmetry, for all firms in S.

3.3.3 Growth

Now that we have shown the existence of a stable symmetric equilibrium in the static model, let us turn to the situation in which the economy grows over time. Specifically, what will happen to the spatial equilibrium when new firms enter the market?

Our conclusion will depend on the relationship between the new firms and those already present. Let us assume, as we did before, that the numbering of firms from 0 to n implies something about the order in which they were created. Let firm 0 be the oldest firm in the economy, using ancient technology and intermediates from other venerable firms. On the other side, let firm n be the latest addition to the spectrum of technologies. We further assume that any new firms that enter the economy will use a technology related to that of the most modern firm. We can then model an increase in the number of firms from n to $n+\delta$ as a continuation of the present structure: as illustrated in figure 3.12 below, the new firms are added at the end and do not change the form of the input-output relationship.

Where will these new firms locate, if we start with the symmetric equilibrium of the previous section? Naturally, they would like to be close to their intermediate-goods suppliers, the majority of whom will reside in region S.

What will happen next? For a while, the symmetric equilibrium will be disturbed: there will be more firms in region S than in region N. We have

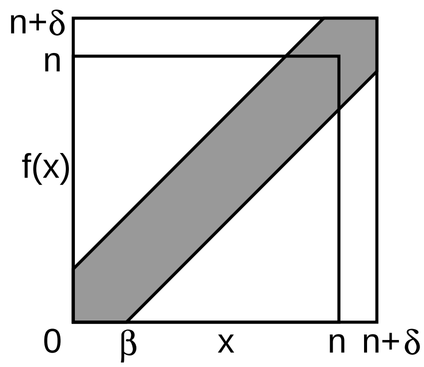

Figure 3.12: Function $f(x)$, defined in formula (3.21), when the number of firms grows from n to $n + \delta$.

seen in previous sections that the mechanism that returns equilibrium is the labor market: increased competition for labor in region S will drive up wages in that region. This will decrease profits for all firms in region S, which leads to the next step in the return to equilibrium: firms will start to think about moving to the N region, where costs are now lower. Which firms will move first? Those with indices close to $n/2$, who already find a large part of their suppliers in the other region, will be the first to find it profitable to go.

When the spatial equilibrium starts changing, where will it end? Without numerical simulations and an explicit rule for relocating between regions we cannot be absolutely sure, but we do know that there is a new symmetric equilibrium close by. Observe that the new input-output function in figure 3.12 is isomorphic to the old function, in figure 3.11 on page 70 above. That is, apart from scale, the supply-demand relations between all the firms in the economy do not change. We can write the grown economy completely in terms of the smaller economy if we substitute $n + \delta$ for n and $n\beta/(n + \delta)$ for β. Thus, from the same proof as in the previous section, we can assert that the symmetric equilibrium is, again, stable.

So we see that when new firms come into existence and settle in region S, the least modern firms in that region can migrate to N to restore the symmetric equilibrium. If this happens, a pattern emerges: there exists a region where new, modern production takes place, and a region where older technologies are used in production. As the economy grows, older firms are relegated to the region where the other old firms are, while new firms settle close to other new firms.

Note that we have not assumed anything about the relative productivity of either type of firm, and that we cannot arrive at any conclusion about the relative well-being of either region. Once the symmetric equilibrium is restored, that is exactly what it is: each region produces and consumes equally much of different, but equally valuable goods.

However, in practice there is often a difference between new and old industry. To be on the growing end of an economy carries some advantages, like a more dynamic labor market, positive external effects on R&D, and the presence of an environment of opportunities. In that sense, this model has a bleak message for the region harboring 'old' firms: even with continuous new entry, it will not escape its predicament through the forces of economic equilibrium. Instead, these forces seem to conspire to keep the difference between the two regions as large as possible.

A point to note is the implicit assumption that we make about the costs of moving a firm to another region. In this model, even though simple trade in final and intermediate products is subject to transaction costs, we assume that moving a firm can be done costlessly. This may be a fair approximation, if we take into account that the decision to move the firm to another region will affect the economic conditions of the enterprise for a long time, making the costs of moving relatively small. Alternatively, we could assume that firms do not actually move, but are subject to bankruptcy when their region becomes overcrowded. Their patent then becomes available and a similar firm re-emerges in the other region, where costs are lower.

If we would make the opposite assumption (large costs of moving) and instead imagine firms sticking to their region no matter what, a process of leapfrogging could emerge in this model. In this process, new firms come to the S-region until wages there have risen to a point where it is more profitable to locate in the N-region and import all intermediate goods. As soon as the first new firm has done this, region N becomes the new center of growth, leading to rising relative wages until the process again reverses. This scenario is harder to model because the different equilibria would be asymmetric, making the analytical solution of the model more involved than it was above. The characteristics of the solution would be very different: each region would see a cyclical movement in its wages and its relative attractiveness to new firms.

3.4 Conclusion

In this chapter, we elaborate on the economic geography model in which ties between firms consist of input-output relationships. When there are positive transport costs, firms will locate close to each other to minimize the costs of their intermediate inputs. A countervailing force is the labor market, where the wage rate rises in regions where many firms cluster, raising again the average costs of inputs.

When we divide firms into sectors, the agglomerating and dispersing forces can be separated. In section 3.2, we show that two groups of firms that do not use each others' products intentionally move away from each other. That is, firms that have no reason to share a region will not do so, because of the labor market pressures of other firms. Instead, they will cluster with firms from their own sector. This behavior can be seen in practice when multinational corporations decide to move manufacturing to low-wage countries. Manufacturing, in these companies, does not need the services offered in industrialized Europe or the US. Raw materials for manufacturing, on the other hand, can be obtained cheaply in low-wage locations.

We look at static equilibria in the first two sections and turn to economic growth in section 3.3. There we assume that technical progress exogenously drives growth, and that ties between firms are stronger when they are created around the same period. It turns out that, for our assumed IO-function, new firms like to enter into the region where other recent arrivals have also located. As time goes by and the economy grows further, other firms in that area are relegated to the a region with older industries. As such, there exists an innovative and a lagging region. Both are equally productive and enjoy equal welfare.

3.A Firm demand and supply

We show the validity of (3.12) by solving it for the price level p. It will turn out that this is the price level that we actually used (which is in formula 3.6).

First we write production as

$$\begin{aligned} z_i &= \left(\frac{L}{n}\right)^{\alpha} Q^{1-\alpha} \\ &= \left(\frac{L}{n}\right)^{\alpha} \left(\frac{L}{n}\frac{1-\alpha}{\alpha}\frac{1}{p_Q}\right)^{1-\alpha} \\ &= \frac{L}{n}\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} p^{\alpha-1} n^{\frac{\alpha-1}{1-\sigma}} \end{aligned} \tag{3.22}$$

where the second step is from (3.7).

Intermediate demand is

$$Qn^{\frac{1}{1-\sigma}} = \frac{L}{n}\frac{1-\alpha}{\alpha}\frac{1}{p_Q}n^{\frac{1}{1-\sigma}}$$

and final demand

$$\begin{aligned} L\frac{y}{p_Q}n^{\frac{\sigma}{1-\sigma}} &= (L+n\pi)\frac{1}{p_Q}n^{\frac{\sigma}{1-\sigma}} \\ &= \left(L+\frac{npz_i}{\sigma}\right)\frac{1}{p_Q}n^{\frac{\sigma}{1-\sigma}} \end{aligned}$$

Adding these two and simplifying, we get total demand

$$\mathrm{TD} = \frac{L}{n}\frac{1}{p\alpha} + \frac{z_i}{\sigma}$$

Into this expression, put (3.12) by requiring that $\mathrm{TD} = z$.

$$\frac{L}{n}\frac{1}{p\alpha} = \frac{\sigma-1}{\sigma}z.$$

We then use the expression from (3.22) for z and simplify:

$$\begin{aligned} \frac{L}{n}\frac{1}{p\alpha} &= \frac{\sigma-1}{\sigma}\frac{L}{n}\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}p^{\alpha-1}n^{\frac{\alpha-1}{1-\sigma}} \Longrightarrow \\ p &= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{\alpha}}(1-\alpha)^{\frac{\alpha-1}{\alpha}}\,\alpha^{-1}n^{\frac{1-\alpha}{\alpha(1-\sigma)}} \end{aligned}$$

Q.E.D.