Chapter 4

Input-output relations in economic geography

4.1 Introduction

An unhindered flow of trade between countries is beneficial to all parties. The most fundamental result of Ricardian trade theory depends on the differing proficiency of different nations in the production of various goods. In addition to the Ricardian gains from trade, Samuelson (2001) reminds us that there exists a *Sraffian Bonus* (Sraffa 1960) when imported goods can be used as intermediate inputs. In a model without costs of transport and with a simple linear production technology, free trade leads to less expensive final products for consumers, but also to cheaper inputs for firms. The latter causes world industry to become more productive when trade is allowed, leaving everybody better off. Like the standard Ricardian gains from trade, this result depends on comparative advantage and holds even when one country dominates the other in absolute productivity in all sectors.

In spite of these theoretical recommendations, not everybody favors free trade. Small, peripheral or underdeveloped countries often choose to close their borders to protect domestic industry. They fear that competition from abroad will be too strong, because foreign competitors are either larger, more established or both. Indeed it does not take much by way of changing the assumptions to throw a spanner in the works of trade theory and diminish the appealing result above. It can be argued that Venables (1996a) does just that. His model has nonzero costs of transport and a nonlinear production technology. There are two sectors, industry and agriculture. Firms in the industrial sector use each other's products as intermediate inputs and because of international price differences in these inputs, a possible outcome of the model is the agglomeration of all industry in one country.

In this chapter, we look at a models that is similar to that of Venables

(1996a). There is an important distinction, however: we allow for more than one industrial sector, and the demand for intermediate inputs can differ between sectors. This more general model will allow us to make observations on the likelihood of complete agglomeration of industrial activity.

4.1.1 Related theory

The model by Venables cited above belongs to the family of Economic Geography models. The central tenet of this body of theory is that agglomeration of activity can be an economic equilibrium, sustained by complementarities that exist in the production process. In the first chapter, we discussed different mechanisms for these complementarities, which formalize the concept of forward and backward *linkages* between firms and the local labor market, or among firms in the same region (These linkages are similar to those discussed by Hirschman, 1958). We briefly repeat the three mechanisms:

- The home market effect, which occurs because firms demand local labor and the local labor force demands the firms' products.

    When both are mobile the interaction of their demands may result in a large local market for both labor and endproducts. In this process, workers choose to move to the agglomerated region which, in turn, becomes an attractive place of business for firms. Models of this type typically rely on a mobile workforce which is tied to its sector, *i.e.,* there are a fixed number of farmers and manufacturing workers. An early example is found in Krugman (1991a).

- Linkages through intermediate goods. When the assumptions on worker mobility are reversed, we obtain a model that applies in an international context. Workers are not allowed to move to a different region, but can choose in which sector they work. Agglomeration can occur in the sense that one region gets all of the industrial activity while the other must import its industrial products. Firms choose to be in the same region because they depend on each other for intermediate goods, the availability of which may offset higher wage costs. This model was developed by Venables (1996a). However, the notion that intermediate goods play a role in the agglomeration process can be traced back all the way to Marshall (1920).

- Finally, we identified a class of models where the agglomeration mechanism is the result of a third sector, where R&D is conducted. This type of model was discussed by Martin and Ottaviano (1996b).

Each of these three classes of models uses the principle of complementarity to explain agglomeration, but specifies a different channel through

which it operates. It depends on the situation that is studied which one of these channels, and thus which one of these models, is most relevant.

In this chapter, we will take a closer look at the second type of economic geography model. Intermediate products are the channel for complementarities here: firms benefit from each other's presence as each producer uses the products of other firms as an intermediate input. When transport costs are nonzero, agglomerations can occur because the advantage of being close to intermediate supply and demand outweighs possible disadvantages, such as a higher wage level. We discuss our choice for this model below.

4.1.2   Intermediate goods, trade, and ties between industries

There are several reasons why the intermediate goods-type of model[1] forms an interesting subject for further research. Its assumptions specify that workers cannot relocate to another region, but *can* change the sector in which they are active. These assumptions are appropriate when we want to study trade between different nations. As Fujita et al. (1999, p. 239-240) show, mobility between different nations may not be zero, but it is close to zero. In the European Union, legislation has been passed to facilitate the movement of workers between the different member states. However, the number of workers who actually migrate to another country is very low. In the year 2001, for instance, only two percent of EU nationals worked in another member country (Migration News 2001). Language problems and "soft barriers," such as differences in pension systems and tax codes, cause the immobility. Meanwhile, it is clear that some European regions are more agglomerated than others: the concentration of activity in the "hot banana" that lies between London, the Ruhrgebiet and Northern Italy[2] is much higher than that in some of the more peripheral European countries. These facts clearly call for a model that features both immobility and agglomeration.

If we want to gather more evidence about the relevance of the Venables model for international trade, we can look at data on flows of trade between industrialized and non-industrialized nations and decide whether its characteristics match the model's predictions. Given that in the model, each firms uses *every* industrial product as an input, it predicts that the flows of trade between industrialized countries contain at least some intermediate products. Final products should flow from industrialized countries to both the periphery and to other industrialized countries. Finally, the periphery

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[1]From now on, we will use the term 'Venables-model' in this text, recognizing the author of the first economic geography-model which used intermediate goods as a channel for complementarities (Venables 1996a).

[2]This area is also known as the "blue banana," after its discovery on a colored map of Europe (ESPON 2003, p. 69).

Imports OECD   Imports non-OECD
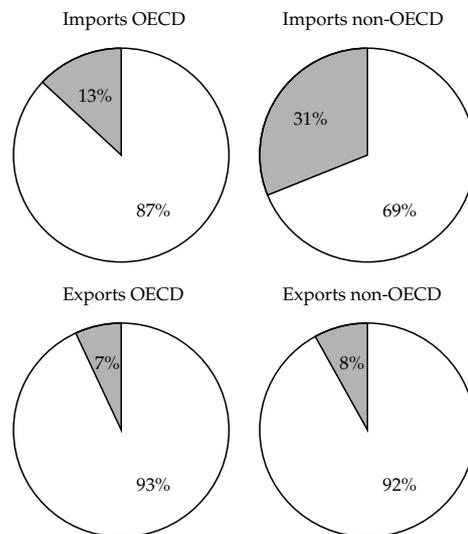
Exports OECD   Exports non-OECD

Figure 4.1: Trade data for Germany in 1999, from and to OECD- and non-OECD countries, from OECD (2000). The smaller, grey slices are the share of 'basic' products (sections 0–4 in the standard international trade classification), while the remaining, white slices are 'industrial' products (sections 5–8). Sections 0 through 4 are, respectively: food and live animals, beverages and tobacco, crude materials (inedible), mineral fuels, lubricants, animal and vegetable oils, fats and waxes. Sections 5 through 8 are chemicals and related products, manufactured goods, machinery and transport equipment.

pays for its imports with non-industrial goods, which should make up the flow of trade from them to the industrialized countries.

To find out about the relevance of the model we could inquire about the accuracy of its predictions. Ideally, we would gather data about trade between a number of regions, agglomerated and peripheral, and find out about the share of final, intermediate and basic, non-industrial products. Unfortunately, data is not gathered using these definitions; we will have to make do with a first-cut approximation, in which we group the more-agglomerated regions and compare them to their less-agglomerated cousins, dividing up the flows of trade in more- and less-basic goods.

An example of such data is in figure 4.1. It shows the characteristics of the imports and exports of Germany in the year 1999. The flows of trade are split into four categories: firstly, we separate out trade with OECD nations, which we take as a rough approximation of trade with industrialized countries, and contrast it to the trade with non-OECD nations. Secondly,

we use a crude categorization of 'basic' and 'industrial' goods, corresponding to different sections of the trade statistics. Our 'industrial' goods will be a proxy for the final and intermediate goods of the model. The 'basic' goods can be produced without industry and correspond to the 'agricultural' sector of the model.

From the figure, we see that our predictions come through in a relative sense: a relatively large share of the goods that Germany imports from countries outside the OECD is basic, compared to the imports from fellow-OECD countries. Most exports are of an industrial nature.[3] But while the data are roughly consistent with the model, there are some differences. We would have predicted imports from non-OECD countries to be all basic or agricultural goods and the three other streams to be largely industrial. In fact, industrial products are a non-negligable part of imports from non-OECD countries. We can identify several reasons for this inconsistency.

First of all, if we assume that the model is true, we might explain the differences between its predictions and the data by measurement error. Our division between OECD- and non-OECD countries does not coincide exactly with agglomerated and non-agglomerated areas. Also, the different sections of the trade statistics that we have used do not exactly match *industrial* and *basic* products in the model's sense. Moreover, some goods such as oil double as both industrial and basic. Finally, because we have measured trade in dollar terms, we can expect the more expensive industrial goods to carry more weight than they would have in terms of weight or volume.

However, at the root of these inconsistency could also be a problem with the model's simplicity. Maybe some industrial firms *did* establish in the non-agglomerated region, where wages are lower. These firms could depend less than average on intermediate products. Because model does not allow for different kinds of industrial firms, this nuance is not a part of its predictions. It is part of this shortcoming that we will try to remedy in the current chapter.

When we look closer at the streams of trade between industrialized countries, we see another shortcoming of the basic Venables model. For instance, look at the data in figure 4.2. It is a breakdown of section 7, or *machinery and transport equipment*-trade between Germany and the Netherlands. Each way, approximately 11 billion US$ worth of goods is shipped. However, the intra-section division is completely different: while Germany exports mostly type 7.8, or *road vehicles*, they import mostly 7.5, or *office machines and computers*.[4]

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[3]It could of course be argued that the large share of industrial goods in the imports from non-OECD countries is an indication that our approximation of Industrial and non-Industrial countries is wrong. While a dedicated researcher could probably find better data, the very point of this exercise is to show that simple notions of countries as being either completely industrialized or devoid of any industry are wrong.

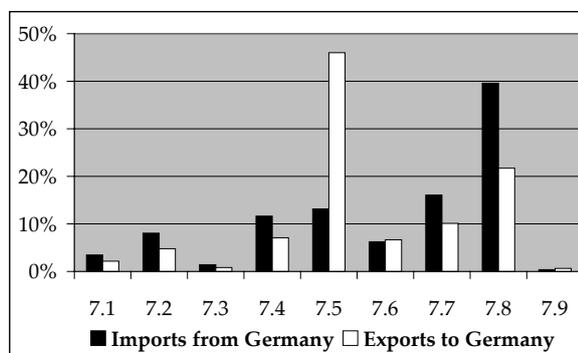[4]Other categories: 7.1: Power generating machinery and equipment, 7.2: Machinery

Figure 4.2: Section 7 trade between the Netherlands and Germany, from OECD (2000). Section 7 is *machinery and transport equipment*. The Netherlands exports mainly 7.5-type goods, *office machines and automatic data processing machines*, to Germany while German exports to the Netherlands contain a large share of section 7.8, *road vehicles*.

This is where the assumption that industry is a homogenous sector again becomes impractical. We know that there exist different sectors, even though in the model they have been lumped together for convenience. Now they turn out to be concentrated in different regions: it appears that firms in the automotive industry are more concentrated in Germany, while the makers of office appliances have concentrated in the Netherlands. This suggests that there is an agglomerating force *within* industries, as well as the force *between* industries that our model predicted.

The existence of such a force is discussed by Krugman (1991a), who gives three reasons for its existence (this is the Marshallian trinity, as they were originally proposed by Marshall 1920). The first two reasons argue that concentration is the effect of specialized labor markets and of externalities. The third explanation, is more in line with the Venables model: the reason for agglomeration within a sector is that inputs *specific to an industry* are available in greater variety and at a lower cost when firms are close together. This is at odds with the assumption that industries are homogenous, since it implies that they use different sets of inputs. Hence, it may be efficient for these industries to agglomerate into different regions.

Indeed, in real life some firms depend on a large amount of intermediate goods and some are less dependent on intermediates, and the types of intermediate goods are known to vary. In the current model, there are only two types of firms, one of which (agriculture) uses no intermediate goods

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specialized for particular industries, 7.3: Metalworking machinery, 7.4: General industrial machinery, 7.6: Telecommunications and sound equipment, 7.7: Electrical machinery, 7.9: Other transport equipment.

while the other uses a bundle comprising all products.

4.1.3   Plan for the chapter

In this chapter, as in chapter 3, we will develop a model of the Venables type. In Section 4.2 below, we will briefly review the Venables (1996a) model and add an extension which allows us to model different industrial sectors. In this chapter, the sectors will be discrete so that it is possible to construct a traditional input-output table that specifies the flows of trade between sectors. We will find that there are several possible equilibria in the extended model, apart from agglomeration and dispersion. In contrast to Krugman and Venables (1996), who assume symmetry between sectors and Fujita et al. (1999), who only discus a small number of special cases, we will to study the effects of different IO-structures in their entirety, concentrating on the boundaries between different types of equilibria.

Section 4.3 discusses the possible types of solutions. We will see that 'catastrophic' changes in the agglomeration of firms can happen when IO-parameters shift marginally. This effect has previously been shown for the transport costs parameter.

Finally, section 4.4 concludes and looks ahead to possible applications of the results.

4.2   The Venables model, and an extension with discrete sectors

4.2.1   One industrial sector

In this section we discuss the Venables (1996a) model with decreasing returns in agriculture. The model is also briefly discussed in chapter 14 of Fujita et al. (1999). A few aspects have been changed to facilitate later extensions, these changes will be noted in the text. In the initial model, we will assume that the world consists of two regions and there are two sectors of production, an agricultural sector and an industrial sector. Later on, we will generalize both the number of regions and the number of sectors.

Both regions are similar in principle, even though their state in equilibrium may be different. Each region has a fixed, immobile labor supply, whose members may choose to work in the agricultural sector or the industrial sector. We describe the production facilities, which are the same in both regions. Total production in the agricultural sector depends only on L_A, the amount of labor used, and is equal to L_A^β ($0 < \beta < 1$). Total agricultural production is divided among all workers in the sector, so that their income equals

$$w = L_A^{\beta-1}. \tag{4.1}$$

In this setup, the wage is greater than the marginal product of labor. This follows because of an implicit assumption that the other production factor, land, is freely available. This way, we do not have to introduce a separate class of landowners. Note that $\beta - 1$ is a negative number, so that the wage rises as the number of farmers drops. Labor is in fixed supply, but can move freely from one sector to another. Therefore, wages are equalized between sectors and the expression in (4.1) is the wage for the entire region. This means that a region with a large industrial sector and small agricultural sector will have relatively high wages.

In the industrial sector, production of a firm i is a function of the applied amounts of labor and intermediate products. Each firm makes a unique product, but all firms share the same production technology; we will therefore omit the subscript i below. The production function for a firm is

$$y = \frac{1}{\phi \cdot \theta_\alpha} L^\alpha Q^{1-\alpha} - F. \tag{4.2}$$

In this function, L is the amount of labor applied and Q is an aggregate of intermediate products. Labor's share α lies between 0 and 1. There are fixed costs F of production which are incurred in the final product. Finally, the constant scaling factor consists of two terms, a positive scalar ϕ and the positive number θ_α, which is equal to $(1-\alpha)^{\alpha-1}\alpha^{-\alpha}$.

From the above production function, it follows that marginal costs are equal to

$$MC = \phi w^\alpha G^{1-\alpha} \tag{4.3}$$

We will normally set $\phi = 1/\theta_\alpha$, so that function 4.2 is of a simple form.

There is a continuum of firms that employ the above production function. As mentioned, their products are unique and we assume limited substitutability between them. All products serve as both final and intermediate. In order to aggregate these products into a single intermediate input Q, we use a method first employed by Ethier (1982): the size of Q is a Dixit-Stiglitz (1977) aggregate of all the different varieties. This amounts to saying that

$$Q = \frac{1}{\psi} \left[nq^{\frac{\sigma-1}{\sigma}} + n^*(q^*)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{4.4}$$

In this equation we introduce our convention to separate the two regions by designating one as *home* and one as *foreign*. We add asterisks to the variables of the *foreign* region. The number of firms, and thus the number of distinct products, in *home* is equal to n, and there are n^* firms in *foreign*. When we bring together a quantity q of each of the n home products and a quantity q^* of each foreign product, formula (4.4) shows that a bundle of intermediate goods of size Q results. This variable plays two roles: firstly, it serves as a measure of intermediate product as in formula (4.2). Secondly, we assume that consumers demand industrial products in bundles

of Q. This means that both firms and consumers have an elasticity of substitution of σ between the different industrial products. As for consumers, we furthermore assume that they spend a fixed fraction μ of their income on industrial products and a fraction $1 - \mu$ on agricultural products. The positive scalar ψ, finally, determines the level of costs of the intermediate bundle. Making it larger means that intermediate goods become more expensive relative to the other factor of production, labor.

We must specify if products of one region are available in the other. We take a dual approach in this matter: agricultural products can move freely at zero cost, but industrial products are subject to iceberg trade costs: only a fraction τ (where $0 < \tau < 1$) of shipped goods arrives—alternatively, the price of goods from another region is $1/\tau$ times the f.o.b. price.

As we saw in section 2.2.2 in the previous chapter, the aggregation in formula (4.4) implies a price index G for the bundle Q, equal to

$$G = \psi \left[np^{1-\sigma} + n^*(\frac{p^*}{\tau})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{4.5}$$

As expected, there holds that an amount E will buy E/G bundles of Q. In formula 4.5, p and p^* are the prices of home and foreign products in their own region. Notice that because of transport costs, the price for *foreign* goods in *home* is higher than p^* by a factor $1/\tau$.

Formula (4.5) implies that firms in the same region set the same price. Knowing that they are in monopolistic competition, all firms employ markup pricing in equilibrium, setting their price at

$$p = \frac{\sigma}{\sigma - 1} w^\alpha G^{1-\alpha}. \tag{4.6}$$

This price is a markup over marginal costs. A similar relationship holds for the foreign region. Notice from (4.5) and (4.6) that there is a circularity between price p and price index G, which cannot be solved analytically. When parameter values are known, however, both G and p may be computed by numerical means.

Clearly, there is a dependence between the two regions, which works as follows. As already stated, labor is tied to its own region. We stipulate that the same holds for the ownership of firms, so that the only interaction between regions takes the form of trade.

As in chapter 1, we have to assume something about the relative speed of adjustment of workers and firms. Workers move between the two sectors in response to wage differences, firms enter or leave the market in response to profits or losses. We will retain our assumption that workers move faster than firms, so that at all times the wage is equal in both sectors, and given by equation (4.1). For firms, the zero-profit condition is $y = (\sigma - 1)F$. We will study the model out-of-equilibrium, so that this condition does not hold everywhere in this chapter.

Optimization by firms implies that labor and intermediate products will be employed such that their marginal costs equal marginal benefits, so that the quantity Q/L can be computed from the relative price w/G. The relative use of intermediate product can be found both in- and out-of the zero-profit equilibrium. When profits are zero, the applied amount of labor is a function of parameters and relative prices:

$$L = \sigma F \alpha \left(\frac{w}{G}\right)^{\alpha-1}. \tag{4.7}$$

Now that we have specified the model, we can think about possible states of equilibrium. We know that each region must have an agricultural sector, because productivity in this sector goes to infinity when its size goes to zero (formula 4.1). At least one region has an industrial sector, as a positive fraction of income is spent on industrial goods no matter what. If both regions have industrial firms, there will be intra-industry trade. As a result of the Armington assumption both firms and consumers will demand domestic and foreign products, though less when trade costs are large.

Because both regions are exactly similar in all aspects, it would appear that a situation where both have the same number of firms might be an equilibrium. Simulation confirms that this, sometimes, is the case. However, the model allows for other solutions as well: for certain sets of parameters, all industry can agglomerate into a single region, leaving the other with only an agricultural sector. It is this result, first obtained in the work of Venables (1996a), that one should be mindful of before opening one's region to trade: it is quite possible that all domestic industry will be lost to the neighbor's industrial core. Thirdly, for a small subspace of parameters, an asymmetrical equilibrium is possible in which both regions have a positive, but different number of industrial firms.

We will use numerical results to show that an agglomerated equilibrium can be stable. It is quite easy, *given that a single industrial core exists*, to compute what it must be like. With $n^* = 0$, expressions such as formula (4.5) become less tangled. We can then compute a solution for all variables.

Using this solution, we evaluate the position of a hypothetical single (small) firm in the foreign region. We used the same method of evaluating the agglomerated equilibrium in section 2.C above, where there was only one sector of industrial firms. This firm would be the only one of its sort, a pioneer breakaway from the core. We can compute the demand that this firm would receive, both from home and abroad, its costs and its profit. The firm would have several advantages over the firms in the agglomerated region: wages are lower and local demand is stronger. Its disadvantage is the high cost of intermediates, which all have to be shipped from the other region. Also, demand from other firms will be lower than it would have been in the agglomerated region. In the balance, these factors determine

the sign of the hypothetical firm's profit. If this potential profit is negative, we conclude that our assumption of zero industry in the foreign region was correct; it would not be profitable to start any. However, if potential profits are positive we must conclude that the agglomeration of industry in the home region is unstable.

Figure 4.3 shows the potential profits of a breakaway firm as a function of transport costs τ.[5] We see that an agglomeration of all firms in one region is stable for intermediate values of τ. This is a well-known result, known as *inverted U dependence*. For very high transport costs, local demand alone is enough to sustain industries in both regions. For very low transport costs, the advantages of agglomeration are outweighed by the disadvantages of using only one workforce. In between, agglomeration is stable.

But how about the other way around? Does the stability of the agglomerated equilibrium automatically preclude a stable symmetric equilibrium? We investigate the symmetric equilibrium as follows. Using numerical methods, it is possible to compute what the symmetric equilibrium would look like, given that it exists. Both regions have an industrial sector, and all firms turn a profit of exactly zero. The regions are completely symmetric: wages, sectoral structure, import and export are the same on either side.

Using the same numerical methods, we can compute the effects of infinitely small changes in the number of firms, allowing wages and prices to adapt but momentarily suspending our assumption of zero profits. This is in line with our earlier assumption that people move faster between sectors than firms move between regions, an assumption that was employed in section 2.C. Thus, we look at the effects of small perturbations in the number of firms: what would happen if an extra firm opened up here, another firm closed there? To capture all possible perturbations, we compute the matrix of derivatives $\partial\Pi/\partial N'$, which is constructed as follows:

$$\partial\Pi/\partial N^T = \begin{pmatrix} \frac{\partial\pi}{\partial n} & \frac{\partial\pi}{\partial n^*} \\ \frac{\partial\pi^*}{\partial n} & \frac{\partial\pi^*}{\partial n^*} \end{pmatrix} \tag{4.8}$$

In this matrix, n is the number of firms in *home* and n^* the number of firms in *foreign*. The vector N stacks both, N^T being its transpose. The vector Π stacks the two profits: each firm in *home* turns an profit of π, the profit of a foreign firm is π^*. In equilibrium, both are zero by definition. Their derivatives with respect to n and n^* can be nonzero, though. From the matrix $\partial\Pi/\partial N$, which contains these derivatives, we can judge the stability of the symmetric equilibrium. Suppose a small increase in n leads to a higher profit for *home* firms, π, and that this was the *only* effect of the increase. Then such a change in n would be self-enforcing: the positive

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[5]Note that transport costs are high when τ is low and small when τ is close to unity. The profits in figure 4.3 are the results of a numerical simulation. The other parameters in example are: $\sigma = 4$, $\beta = 0.8$, $\alpha = .37$, $\mu = 0.8$, $\psi = 1$ and $F = 0.001$.
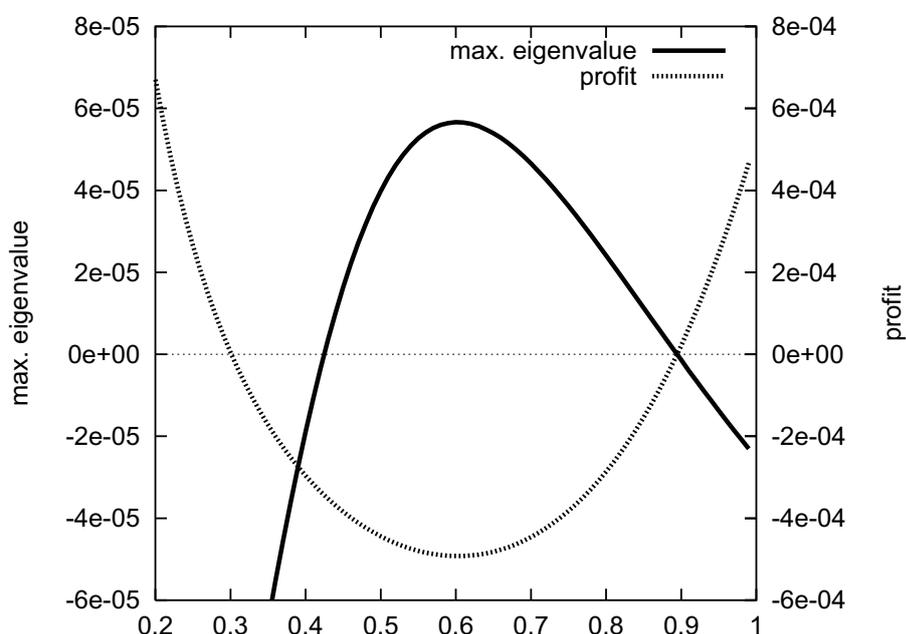
Figure 4.3: This figure shows two lines that indicate the stability of two equilibria as a function of transport costs τ (τ is on the horizontal axis). The broken line pertains to the agglomerated equilibrium. On the right-hand y-axis, it shows the potential profit of a single firm in the other region. If this is negative, the equilibrium is stable. The continuous line uses the left-hand axis and pertains to the symmetric equilibrium. It shows the maximum eigenvalue of the matrix $\partial\Pi/\partial n$, the derivative of profits to the number of firms. If there exists a positive eigenvalue, the symmetric equilibrium is unstable. Note that the lines cross the x-axis at the same point on the high-τ side, but not on the other side.

profits would draw in more firms, which would in turn cause even higher profits, *et cetera*. Similarly, a fall in the number of firms would drive profits below zero. This would encourage more firms to exit, causing profits to fall further. We can conclude that if the upper left element of the matrix is positive, and the rest are zero, the equilibrium is unstable.

By analogous reasoning, we can see that a negative upper-left element and surrounding zeros would make the equilibrium stable. However, the matrix of derivatives is rarely this simple. Instead we must generalize the above approach and examine the eigenvalues of the matrix $\partial\Pi/\partial N$. Suppose there exists an eigenvalue greater than zero. Now suppose that a perturbation to the number of firms, $\partial(n, n^*)$ occurs that just happens to be a multiple of the corresponding eigenvector. The resulting changes in profits $\partial(\pi, \pi^*)$ would have the same direction as the change in the number of

firms, $\partial(n, n^*)$. This change would be reinforcing, as it was in the scalar example above. It appears that the presence of one of more positive eigenvalues signals that the symmetric equilibrium is unstable.

This assertion can be made more rigorous. There presumably exists a relationship between profits and the number of firms which we have left implicit, stating only that firms enter or exit until profits are zero. Suppose for a moment that this relationship is linear, say,

$$\dot{N} = \frac{\partial N}{\partial t} = a \cdot \Pi \tag{4.9}$$

with a a positive scalar and N and Π vectors, as above. Now consider the equilibrium numbers of firms $\tilde{N}$ and the variable $N' = N - \tilde{N}$. The equilibrium value of N' is zero.

Now we linearize our model around $\tilde{N}$. By (4.9), there holds that

$$\begin{aligned} \dot{N}' &= a \cdot \Pi \\ &\approx a \cdot \frac{\partial \Pi}{\partial N^T} \cdot N' \end{aligned}$$

This is a homogeneous linear differential equation in N', whose stability properties are determined by the matrix $a \cdot \partial \Pi / \partial N^T$. If the real parts of its eigenvalues are all negative, the solution is asymptotically stable (see, for instance, Brock and Malliaris 1989, p. 66). As the sign of the eigenvalues does not change because of the positive scalar a, the original result still holds.

How does this analysis relate to the one-dimensional computations of break- and sustain values of transport costs that was introduced in section 2.C? Recall that we made a number of assumptions in that section to calculate the break-point, the value of transport costs at which the symmetric equilibrium breaks down: first of all we started with a symmetric equilibrium, and we assumed that changes in the number of firms in the two regions were opposite and of equal size. That is, we studied a perturbation in the number of firms in one region, and simultaneously looked at the opposite perturbation in the other region. Because there was only one sector of industrial firms, the perturbation took place along a single dimension. This made the analysis easy to handle, as only one derivative had to be computed. The fact that the equilibrium was symmetric allowed us to simplify the model to the point where an analytical solution to the derivative could be found.

In the current setup, the increased complexity of the model makes it much harder to arrive at a similar simplification. For one, there may be more than one sector of industrial firms and the sizes of the different sectors do not have to be symmetric. Furthermore, if we want to rule out destabilizing changes in the number of firms, we now have to look at all

possible changes in a higher-dimensional space. With two sectors, for instance, the equilibrium may be stable with respect to an increase in the number of firms in both sectors, but unstable against an increase in one, and a decrease in the other sector. This calls for a more general method, in which all possible changes in the number of firms are analyzed at once. An added benefit is that we no longer need to concentrate on symmetric changes only.

Using the eigenvalues of the matrix in (4.8), we can quickly identify destabilizing perturbations. However, this more general analysis has the drawback that we rely on a numerical approximation to the derivative, where previously an analytical solution could be found. This has to do with the increased complexity of the model, where now even a symmetrical equilibrium can be less than straightforward as the relative number of firms in the two sectors depends on the value of the input-output matrix. This precludes a sufficient simplification of the problem.

In figure 4.3, we have plotted the maximum eigenvalue for the matrix $\partial\Pi/\partial N$ (given that there is a symmetric equilibrium) for the same range of transport costs as before.[6] We see that on the high end of τ, the symmetric equilibrium is unstable for exactly the same range of values for which the asymmetric equilibrium was stable, and *vice versa*. On the low end of τ, there exists a region where both equilibria are stable. In this part, history decides which equilibrium attains. We conclude that in general, the symmetric equilibrium is stable at the high and low transport costs, but not in between.

4.2.2   Types and determinants of equilibrium

The equilibrium that attains in this model is the result of opposing forces of agglomeration and dispersion. A force of agglomeration is the use of intermediate products that are subject to transport costs: when dependent on these products, it pays to be close to their suppliers. A force of dispersion are the scarce laborers: when many firms pack into one region, wages will go up and settling elsewhere becomes more attractive. We discuss the factors that affect the balance between these forces and look at the equilibria that result.

Firstly, to illustrate the dispersive effect of elastic wages, we examine the counterexample $\beta = 1$. In this configuration there are constant returns in agriculture and wages are equal to unity in both regions regardless of the whereabouts of the industrial sector. The results of a simulation with this model are in figure 4.4, whose setup is similar to figure 4.3 on page 90.[7]

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[6]As before, these values are the result of a numerical simulation, in which the Hessian matrix 4.8 is approximated.

[7]In the simulation of figure 4.4, we have set the share of consumer income spent on manufactures (μ) to 0.4. This way, the industrial sector is small enough to agglomerate into
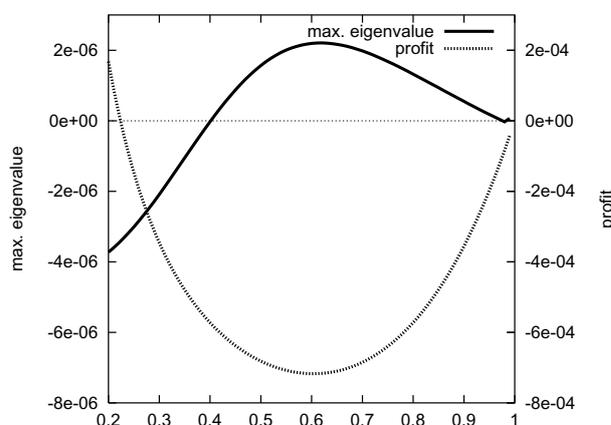
Figure 4.4: The stability of two types of equilibrium as a function of transport costs τ (horizontal axis). See figure 4.3 for details. In this simulation, $\beta = 1$ so that wages are constant. The share spent on manufactures, μ, is set equal to 0.4.

We see that the maximum eigenvalue of the matrix $\partial\Pi/\partial N$ is larger than zero for almost all values of τ, indicating that the symmetric equilibrium is unstable. At the same time, the profits of a firm that would defy the agglomerated equilibrium are negative for a large range of τ, indicating that total agglomeration is a stable outcome. These results confirm that elastic wages form a force of dispersion: without them, agglomeration is almost inevitable, as there are no disadvantages to clustering into one region.

After we reinstate decreasing returns in agriculture, it stands to reason that the relative importance of the factors labor and intermediate products will be an important determinant of the type of equilibrium that is found. In our model, a measure of this relative importance is the parameter ψ. This can be seen from formula 4.5: when ψ is high, productivity in the making of intermediate goods is lower, rendering them more expensive. This causes producers to shift to labor as an input factor and thus increases the effect of wage differences.

Using equations 4.5 and 4.3, we can compute how ψ factors into marginal costs. Combining the two expressions shows that levels marginal costs are proportional to $\psi^{1-\alpha}$. This factor serves to magnify the effect of w. A high value of ψ means that agglomeration, and its accompanying wage difference, become less likely. A low value of ψ means that wage differences become less important and can be overcome in favor of agglomerative forces.

A number of simulations where different values of ψ are used can be

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one region. The original, larger value of μ would have caused expulsion of agriculture from one of the two regions, presumably driving up the wage after all. For clarity, we avoid this complication.

seen in figures 4.5 through 4.7. In the first figure, we have set ψ to 0.1, making intermediate goods cheaper relative to labor. The change shows up mainly in the level of the maximum eigenvalue of $\partial\Pi/\partial N$, which is much lower than before. Interestingly, the sign of the maximum eigenvalue as a function of τ hardly changes, leaving the relation between transport costs and equilibrium almost the same as in figure 4.3. It appears than although the incentive to move away from a symmetric equilibrium is smaller, it is still *positive*.

Things are entirely different in figures 4.6 and 4.7, where $\psi = 2.8$ and $\psi = 20$, respectively. Making intermediate products much more expensive enhances the dispersive power of wages to the point that only for a very small portion of τ-space, agglomeration is stable and dispersion is unstable.

So far, we have talked only about complete agglomeration and complete symmetry, although we mentioned a third possible equilibrium. That type of equilibrium occurs for small subset of all possible combinations of ψ and τ. Observe that in figure 4.6, where $\psi = 2.8$, for τ just below 0.8 both lines lie *above* the x-axis, indicating that both the symmetric and agglomerated equilibrium are unstable. Figure 4.8 shows a close-up of that part of the τ-axis. In this case, we find that the only stable equilibrium is one where both regions have some industry, although one region has a smaller number of firms than the other.

The third equilibrium, which we will call the 'overflow' equilibrium[8] plays a role when we look at situation where transport costs steadily decrease. A world where τ becomes larger, ultimately reaching unity, was first discussed by Krugman and Venables (1995). They showed that in such a scenario, the equilibrium will jump from one state to another, as stability changes. The possibility of an overflow equilibrium as an intermediate stage between agglomeration and symmetry precludes such jumps. The overflow equilibrium only occurs for certain values of ψ, though.

As a theoretical result, the inverted-U dependence on transport costs is both surprising and useful. It shows that the economic geography-type of models can be applied in the context of international trade. It also allows the broad insights that are given in Krugman and Venables (1995), among others. However, as a tool for empirical analysis, the model is too coarse. It assumes two sectors, agriculture and industry, where the latter is completely homogeneous. This assumption does not do justice to the complicated relations that often exist between different firms in the 'developed part' of an economy. To understand the complexities of relations between different countries, we must be able to characterize industries as upstream or downstream, for instance. A natural extension to the model would therefore be to specify the input-output relations that exist between

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[8]In the overflow equilibrium, the agglomerated region lets some of its firms flow into the agricultural region, but remains the dominant seat of the industrial sector.
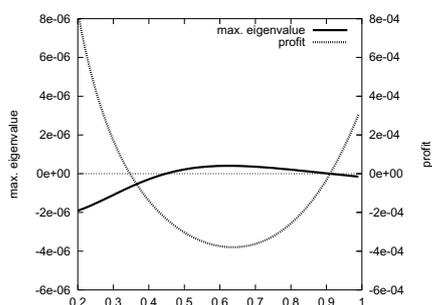
Figure 4.5: Stability of two types of equilibrium as a function of τ, with $\psi = 0.1$. This makes labor much more expensive than intermediate products.
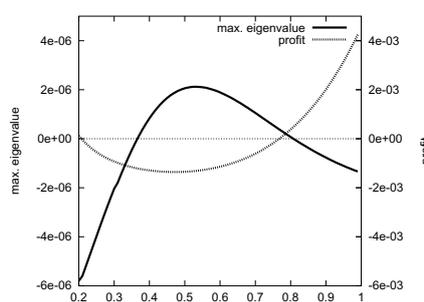

Figure 4.6: $\psi = 2.8$. Intermediate products are more expensive than labor. Note that there is an area where both equilibria are unstable, close to $\tau = 0.8$. See figure 4.8 below.
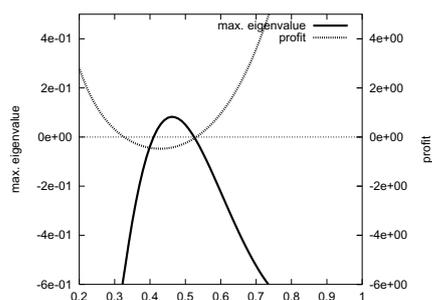

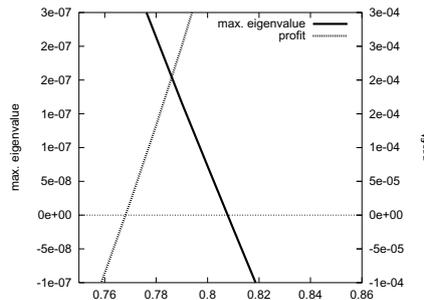Figure 4.7: $\psi = 20$. Intermediate products are much more expensive than labor.

Figure 4.8: $\psi = 2.8$. A closeup look at figure 4.6, which reveals that there exist values of τ where both equilibria are unstable.

different industries in an input-output (or IO) matrix.

We shall use the term *IO matrix* to refer to the set of parameters that indicate how intermediate products from different sectors enter the production function of the various firms. It has a natural empirical counterpart in the *IO table* of an economy. This table, which is regularly constructed for all major economies, specifies the volume of trade between the different sectors. As such, it is an indication of the strength of linkages between those sectors, given that these linkages work through the trade in intermediate goods.

The extension of the model in Venables (1996a) with an IO matrix is taken up in a number of papers, including Krugman and Venables (1996) and Venables (2000). A useful summary of the results is given in chapters 15

and 16 of Fujita et al. (1999). Their main results are two. Firstly, if you assume a form of labor-augmenting technological growth, plus a number of sectors connected by a fairly general IO matrix, an interesting growth process follows. In the beginning, all the industry is agglomerated in one region. Once this region is too small to hold all industry, some sectors make the jump and agglomerate in the second region. This pattern continues, and it suggests a mechanism in which the growth process is punctuated by sudden changes in the economic structure. Differences in the IO matrices are kept to a minimum—the analysis shows that upstream sectors are the first to leave a region, as are those with the weakest links to other industries.

A second main result is that in a model with two regions, no growth and only industrial production, there are two possible equilibria, depending on the costs of transportation. The two sectors can either both choose to settle in both regions, leading to a mixed equilibrium, or the regions can become specialized, each being the host to only one sector. This model is used to explain the fact that many industries in the US are concentrated, while the same industries appear in many countries in the EU. The authors show that this phenomenon can be traced back to lower costs of transportation in the New World.

4.2.3   A model with discrete sectors

With this model, we will try to get some insight into the different types of equilibrium that obtain when we vary the IO matrix. In order to limit the possible number of equilibria and keep the analysis manageable, we only look at the simplest possible setup: a situation where there are two industrial sectors and two regions. This will allow us to present the results in a graphical manner later on. An extension to more sectors is straightforward and pursued in chapter 15 of Fujita et al. (1999), among others. We have made a different generalization in chapter 3, where we discussed a continuous IO structure.

We assume two regions and two kinds of firms, agricultural and industrial. The industrial firms are divided into two sectors. The products of all the different firms are consumed in both regions by agents who maximize utility,

$$U = A^{1-\mu_1-\mu_2} Q_1^{\mu_1} Q_2^{\mu_2} \tag{4.10}$$

where A is consumption of the agricultural good and Q_i is the consumption of products of (industrial) sector i ($i = 1, 2$). As before, we assume that agricultural good is homogeneous and freely tradeable across regions. It will serve as the numéraire. As follows from (4.10), the agricultural sector receives a fixed fraction $1 - \mu_1 - \mu_2$ of each agent's income.

The industrial goods are heterogenous again, and each sector is subject

to monopolistic competition. The aggregation goes according to

$$Q_i = \left(n q_i^{\frac{\sigma-1}{\sigma}} + n^*(q_i^*)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \qquad (4.11)$$

for $i = 1, 2$. This equation is similar to (4.4), but it now operates on the sectoral level. Notice that we assume that the values of σ are equal for the two sectors. This is not essential to the results, but does simplify the analysis considerably. Notice also that, compared to (4.4), we have left out the scaling parameter ψ. This parameter, which is meant to vary the level of costs of intermediate goods, will be reintroduced at the appropriate level below.

The different Q_i's are themselves aggregated by consumers (as in formula 4.10 above) and serve as bundles of intermediate products. The aggregation of Q_i's into a factor of production is sector-specific and follows from each sector's production function. This function is

$$y_i = \frac{1}{\phi \cdot \theta_\alpha} L_i^\alpha \left[\frac{1}{\psi_i \cdot \theta_{\eta,i}} Q_{i,1}^{\eta_i} Q_{i,2}^{1-\eta_i} \right]^{1-\alpha} - F. \qquad (4.12)$$

This function is similar to (4.2): y_i is the production of a firm in sector i.[9] It uses labor and bundles of intermediate product $Q_{i,k}$ with $k = 1, 2$ the supplying sector. The bundles are from formula (4.11); we again assume that the elasticity of substitution between products of different producers in the same sector, σ, is the same for final and intermediate demand. Knowing this, when it comes to pricesetting the producer does not have to worry about the different clients and can set the same price for all: the usual markup over marginal costs, $\text{MC} \cdot \sigma/(\sigma - 1)$. Finally, each firm faces a fixed cost F that is paid in the final product.

There are a number of constants in formula (4.12). The θ's are defined as follows:

$$\begin{aligned}
\theta_\alpha &= \alpha^{-\alpha} \cdot (1-\alpha)^{\alpha-1}, \\
\theta_{\eta,i} &= \eta_i^{-\eta_i} \cdot (1-\eta_i)^{\eta_i-1},
\end{aligned}$$

and serve to normalize associated costs. The marginal costs for a firm in sector i in the *home* region are

$$\begin{aligned}
MC_i &= \phi \cdot w^\alpha \cdot (\tilde{G}_i)^{1-\alpha}, & (4.13) \\
\tilde{G}_i &= \psi_i \cdot G_1^{\eta_i} \cdot G_2^{1-\eta_i}, & (4.14) \\
G_i &= \left(n p_i^{1-\sigma} + n^* \left(\frac{p_i^*}{\tau} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. & (4.15)
\end{aligned}$$

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[9] Notice that we do not index by region—it is assumed that the production functions are similar in both regions.

These expressions follow directly from (4.12). Note that G_i is the price index of goods from sector i in the *home* region, and $\tilde{G}_i$ is the price index of intermediate goods, *used by sector i*, in the *home* region. For the *foreign* region, G_i^* and $\tilde{G}_i^*$ could be defined.

From (4.13)-(4.15), we see that marginal costs are a weighted geometrical average of wage costs in the *home* region and the price of intermediate goods. The latter consist in turn of a weighted average of the price indices of goods from sectors 1 and 2. The price index of goods from sector i, finally, is a weighted average of all prices in the sector, both of firms in the *home* region and in the *foreign* region. Note that in order to use prices from the other region, we have to take the transport costs into account.

From the coefficients η_i in formula (4.12) we can construct a two-by-two IO matrix,

$$IO = \left[\begin{array}{cc} \eta_1 & \eta_2 \\ 1 - \eta_1 & 1 - \eta_2 \end{array} \right]. \tag{4.16}$$

We defined an IO matrix as containing the shares of the budget for intermediates that go to the different sectors. The columns sum to one, indicating that the total budget for intermediates is exhausted. This matrix can be constructed from an IO table by dividing the entries (the flow of trade from one sector to another) by their column sums. Thus, for instance, η_1 is the share of their budget for intermediate goods that firms in sector 1 spend on products from their own sector.

So far, we have left the scaling constants ϕ and ψ unspecified. As before, we will set $\phi = 1/\theta_\alpha$ for a simplification of (4.12). For the same reason, we could set $\psi_i = 1/\theta_{\eta,i}$ for all i for a baseline result and compare cases where different values of ψ lead to a different outcome. However, another issue comes to the front, which is the result of our assumption that discrete sectors are aggregated using a Cobb-Douglas function (as in 4.10). First of all, we need to recognize that the precise categorization introduced into the model is often the result of a judgement call, affected by factors such as data availability. Depending on data, we could carve the economy into two sectors or into twenty. If we want to compare the results of the two-sector model to those in a twenty-sector model, they should at least be the same for some special twenty-sector cases.

A natural point of departure is a generalization of the one-sector model of section 4.2 into an s-sector model ($s \geq 2$) with the IO matrix $\eta_{i,j} = 1/s$ for all i, j. In this case, intermediate bundles consist of equal amounts of products from *each* producer, regardless of the number of sectors. It seems natural that the price index of intermediates $\tilde{G}$ (which is equal across sectors) should be the same for any value of s. Whether the intermediate goods come from two sectors or from twenty seems more of an administrative concern than something that would influence the price of that bundle.

In order to achieve such equivalence, it is important to remember that

the monopolistic competition that exists between firms does not cross the sector boundary. Firms within the same sector are monopolistic competitors, but each sector in total is guaranteed a fixed share of each budget spent on industrial goods. The latter is a result of our various Cobb-Douglas assumptions. Therefore, the love-of-variety effect that causes increasing returns to the number of firms, only works on the number of firms *within a sector*. In this model, carving the economy up into many sectors thus has the undesirable effect of reducing overall efficiency.[10]

This can be seen as follows: suppose we convert a single-sector economy into an s-sector economy, with all entries of the IO matrix equal to $1/s$. The price index of intermediate goods in each case is

$$
\begin{aligned}
\tilde{G} &= \left[n \cdot p^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\
&= n^{\frac{1}{1-\sigma}} \cdot p & (4.17) \\
\tilde{G}_s &= \left[\frac{n}{s} \cdot p^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\
&= \tilde{G} \cdot \left[\frac{1}{s} \right]^{\frac{1}{1-\sigma}} & (4.18)
\end{aligned}
$$

Here, $\tilde{G}$ is the one-sector price index and $\tilde{G}_s$ is the s-sector price index of a bundle of intermediates. As s is larger than one, $\tilde{G} < \tilde{G}_s$, or, the choice of the number of sectors s affects the price of intermediates. In order to preclude this result we introduce s, the number of sectors, into ψ_i: below, we set

$$
\psi_i = \frac{s^{\frac{1}{1-\sigma}}}{\theta_{\eta,i}}. \tag{4.19}
$$

This way, results between different categorizations are comparable in principle.

The model is now operational. We can compute the equilibrium for a region, given the number of firms and the prices of goods in the other region, and given the demand from the other region for home products. A detailed description of the solution method is provided in the next paragraph.

4.2.4 Solving the model

We arrive at the numerical solution of our model in different stages. The order in which these stages are computed reflects our assumptions about the speed of movement: because we think that workers shift sectors faster than firms can enter or exit, we solve the model in this order, starting with

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[10]In effect, we have assumed that there exists a large payoff to variety within a sector but not between them. An alternative approach, in which this problem does not occur, was discussed in the previous chapter.

an initial guess for N, the numbers of firms and and L, the sector-specific numbers of workers per firm:

- Given N and L, we compute regional wages, prices of intermediates and prices for each type of product. The latter two have to be solved simultaneously.

- Given wages and prices, we compute the demand for each type of good. This leads to profits, which are distributed among workers in the same region as the firm (we assume local ownership). Profits again lead to extra demand, so the two have to be solved simultaneously.

- With demand and supply for each firm thus computed, we change L to correct any imbalances between supply and demand. This brings us back to the first bullet point. L is changed iteratively in a numerical Gauss-Newton procedure.

- Now that demand equals supply and wages balance the labor market, we look at profits. Positive profits lead to entry, losses lead to exit. This changes N and brings us back, again, to the first bullet point.

- The routine stops when profits or the corresponding number of firms are zero. For N as well, a Gauss-Newton procedure is used.

We add restrictions. By indicating which sector-region combinations should have no firms, or equal numbers of firms, we study the different equilibria.

4.3 Types of solutions

4.3.1 A Taxonomy

We now look at possible equilibrium results of our model. The results depend very much on the share of industrial products in consumption, $\mu_1 + \mu_2$. If it is larger than $1/2$, the manufacturing sector has to be spread over two locations. If it is smaller than $1/2$, the sector can agglomerate into one region. When we assume that $\beta < 1$, so that there are decreasing returns in agriculture, four types of equilibrium can occur:

1. All industrial activity agglomerates in one region. This happens when $\mu_1 + \mu_2 < 1/2$, the elasticity of substitution σ is small and firms from different sectors use each other's products as intermediary inputs. The region without industry imports industrial products for final consumption, and trades them for agricultural products. In the figures below, this solution is indicated as DEV (for a *developed* region versus an undeveloped region.) The number of firms in each sector is

determined by demand, both final and intermediate. This means that the sectors need not have the same number of firms, as they do in the figure.

2. Both regions get exactly the same, positive number of firms in each sector. This happens when the share of industrial products in consumption is high, and firms from different sectors use each other's products as intermediates. There is only intra-industry trade in equilibrium, with no trade in agricultural products. The values of μ_i do not matter. In the figures below this solution is indicated as SYM, indicating a symmetric equilibrium. Note that the sectors are not necessarily symmetric to each other, as the number of firms is once again determined by intermediate and final demand.

3. Each region gets only one of the two industries. This happens when firms in one sector use very little of the other sector's products as intermediate inputs and trade costs are reasonably low. In equilibrium, there is only inter-industry trade. This solution is indicated as AGL, the outcome where each industry agglomerates. The number of firms in each region depends on the demand for its sector.

4. Finally, in the fourth type of equilibrium, one region specializes completely in a particular sector, say sector 2. The firms from sector 1 agglomerate in the other region, but demand for sector 2 goods is so large that it cannot be filled by the firms in the specialized region. Thus, the some sector 2-firms also appear in the other region as well. The relative number of firms in each region depends on the size of demand, but the sector that has firms in both regions is unevenly distributed. This solution occurs only when $\mu_1 + \mu_2 > 1/2$, and is indicated as OVF, for overflow.

Equilibria DEV and SYM are the results that Venables (1996a) found. For intermediate values of transport costs, all of industry agglomerates into one region. This is equilibrium DEV. For extremely low and high values of transport costs, equilibrium SYM obtains.

The AGL equilibrium was found by Krugman and Venables (1996), in a model without agriculture. The separation of industries occurs when the links between the different sectors are weak, but they are strong within the sectors. The equilibrium obtains under small values of the costs of transport.

The fourth type equilibrium (OVF) has, to our knowledge, not been discussed in the literature. It is an interesting type, where most of industry agglomerates into one region, but a small number of firms finds it profitable to settle in the agricultural region. It bears resemblance to the equilibrium discussed in figure 4.8, in the one-sector case.
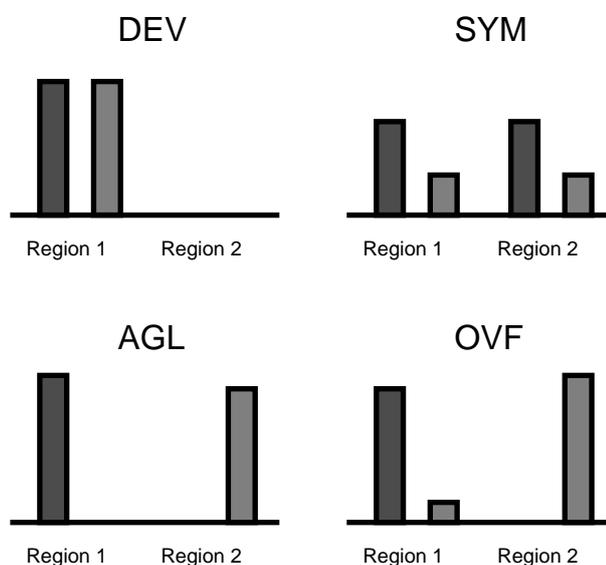
Figure 4.9: The four types of equilibrium, stylized. We look at two sectors, two regions. The graphs show the two regions left and right, and the number of firms in the two sectors is indicated by the height of the two bars.

Figure 4.9 shows a stylized impression of the four types of equilibrium for future reference.

4.3.2   Stability of equilibria

Given that the initial conditions play a role in the final equilibrium of the model, we have to characterize the results of a particular set of parameters in terms of stability, as we did before. This means that we cannot predict whether a certain IO matrix will produce a separation between the industries or a mix of both sectors in both regions.

The parameters of the model are chosen with care. We take the elasticity of substitution $\sigma = 7$, a rather low value that is often chosen because it makes it hard for industries to substitute between different intermediate inputs. This way, the linkages are strong and we are able to see some interesting results. The size of the manufacturing sector, $\mu_1 + \mu_2$ is taken equal to 0.9, so that manufacturing is too big to fit in one region (we look at the other case below). This way, we cannot find the total agglomeration result (type DEV). The other parameter values are fixed costs $F = .008$, labor share $\alpha = 0.6$ and the parameter that governs agricultural decreasing returns, $\beta = 0.9$. Transport costs, finally, are set to $\tau = 0.9$.

The results of this simulation are in figure 4.10. These are the stability characteristics for each possible IO matrix. The figure is laid out as follows: the value of η_1 (see formula 4.16) is on the horizontal axis and the value
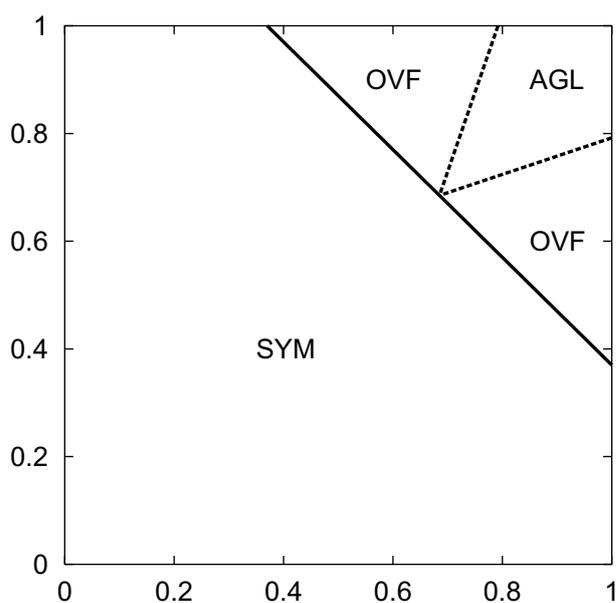
Figure 4.10: Each point in this square represents a 2-by-2 IO-matrix. The upper-left entry of the matrix (the share of sector-1 products in sector 1's intermediate good) is on the vertical axis. On the horizontal axis is the bottom-right entry (the share of sector-2 products in sector 2's intermediate good). Because these two entries automatically define the remaining two, the square represents all possible 2-by-2 IO matrices. There are three possible equilibria in the model that correspond to the indicated areas in this figure. SYM is the symmetric equilibrium and AGL is a complete separation of the two sectors whereby each agglomerates in its own region. Finally, OVF is the so-called 'overflow' equilibrium.

of $1 - \eta_2$ is on the vertical axis. Any point on the square thus defines a
certain IO matrix, and the whole square represents the set of all possible IO
matrices.

We indicate where each of the four different types of equilibrium is sta-
ble, using the abbreviations introduced above. As it turns out, the areas of
stability are mutually exclusive. This can be seen in figure 4.11, which is
similar to figure 4.3 in the sense that on the vertical axis, we see the max-
imum eigenvalue and the profit of a breakaway firm. On the horizontal
axis, however, are not transport costs τ (those are constant) but is η_1, which
for this graph is equal to $1 - \eta_2$. That is, the graphs shows what happens
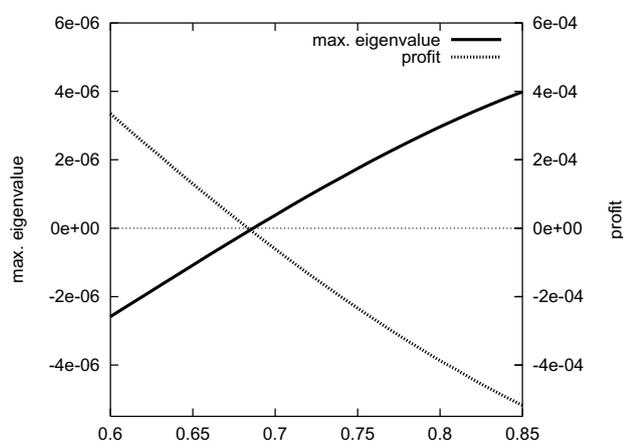along a diagonal line in figure 4.10

Figure 4.11: A trip along the diagonal of the 'map figure'

Is this pattern of stability typical for all parameter values in this model?
We present another outcome in figure 4.12, where the parameters have been
changed to $\sigma = 4$, $\tau = 0.7$, $\alpha = 0.37$ and $F = 0.001$. These are the same as
in the one-sector model that was discussed using figure 4.3. Notice that the
OVF equilibrium no longer obtains. The equilibrium in which two groups
of firms each agglomerate in their own region is stable for a wider range of
parameters. Elsewhere, we find a symmetric outcome.

4.4   Conclusions

In this chapter, we again looked at a Venables-type economic geography
model where the industrial firms are divided up into different sectors. In a
departure from the assumptions of chapter 3, we opted for a discrete num-
ber of sectors. Therefore, we can identify groups of firms who share the
same characteristics with respect to their intermediate goods preferences.

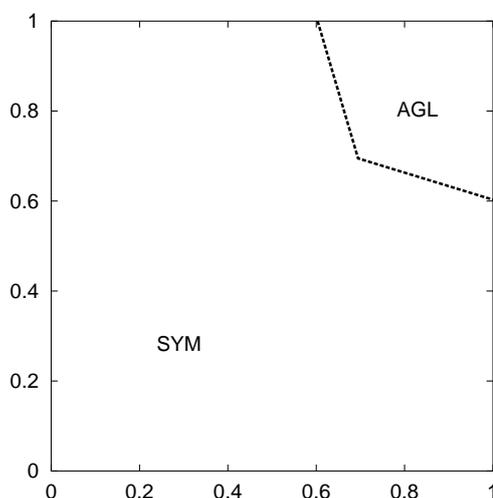How will these groups of firms spread out over different regions, given

Figure 4.12: Each point in this square represents a 2-by-2 IO-matrix. The upper-left entry of the matrix (the share of sector-1 products in sector 1's intermediate good) is on the vertical axis. On the horizontal axis is the bottom-right entry (the share of sector-2 products in sector 2's intermediate good). In this case, $\sigma = 4$, $\tau = 0.7$, $\alpha = 0.37$ and $F = 0.001$, as in figure 4.3.

that there are costs of transport between the two regions? In chapter 3, we saw that simple cases yield simple answers: when two groups of firms only use each other's products as intermediate inputs, these groups will each cluster in a different region. That way, they profit from each other's closeness while staying away from the negative externalities of other firms, such as a higher wage rate.

In this chapter, we were able to take this model further, looking at the all the possible degrees to which to sectors can be intertwined. That is, we looked at all possible input-output matrices between two sectors and mapped the type of spatial equilibrium that obtained on the space of IO-structures.

The simple cases of chapter 3 can be reproduced in this setting: we see them in the right-top and left-bottom corners of figure 4.10. The right-top corner is the case where two groups of firms only use inputs from their own group. Each group will agglomerate in one region. In the left-bottom case firms in each group only use output from the *other* group as intermediates. In this case, each region will have a 50-50 mixture of firms from each of the two groups.

What is interesting is the middle ground. The rest of figure 4.10 shows how the borders between the different equilibria lie. We see (figure 4.11) that there is a clear point where the stability of the different equilibria switches. This means that a small change in technology, whereby a parameter of the IO matrix changes, can cause a dramatic change in the type of equilibrium that obtains. We have seen a similar disproportionate effect of a changing parameter before: a small change in transport costs can switch the model's equilibrium as well (see figure 4.3, for instance).

This chapter has thus demonstrated the dependence of model outcomes on IO parameters, and shown that a small change in these parameters can change the outcome dramatically. But it has also shown a useful way of computing the equilibrium of a model with a number of discrete sectors, when those sectors have ties of different strength, summarized in the IO matrix. If we can find relevant parameters for such a model, we can use it to compute counterfactuals. This would make the model into a tool for policy evaluation. We start work on finding the relevant parameters in chapter 5 below. Using the results of that chapter, we will construct a model of the Dutch economy in chapter 6, in which there are fourteen different sectors. This model will then be used in a policy evaluation exercise.