

# Analysis of Monetary Policy Rules

## The NAKE lectures of Michael Woodford

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### 1 Introduction

The lectures that professor Woodford taught at the recent Wageningen version of the NAKE workshop were, in essence, about monetary policy rules. But rather than taking a partial approach, they involved the construction of a complete macro model and submerged the crowd of listeners in the rigorous Rotemberg–Woodford methodology (as also featured in Rotemberg and Woodford, 1992, for example).

Monetary policy rules describe the behavior of a central bank’s monetary policy, either *ex ante* as a policy announcement or *ex post* as a rationalization of the bank’s actions by economists. The current interest in policy rules is from the side of central bankers as well as academia. The central bank needs an explicit conceptual framework because of increased autonomy, and because many of the old guideposts, such as monetary targets and exchange rate goals, have become obsolete. Also, precommitment to a rule may be a better way to implement a policy than haphazardly fixing the rate because of the way in which consumer expectations are affected.

In academics, the *communis opinio* now is that money matters, not so much as the instigator of shocks, but more in terms of the propulsion of exogenous shocks to the economy. Developments in modelling now allow for a micro-founded analysis of this propulsion that was not possible in the past. Those analyses often call for the formulation of consumers’ expectations about monetary policy, which are difficult to formulate without a rule.

A well-known example of a monetary policy rule is by Taylor (1993), who gives the desired interest rate as a function of the steady-state interest and inflation rate ( $r^*$  and  $\pi^*$ ), trend output  $y^*$  and the actual realizations of these variables:

$$r_t = r^* + \pi^* + \phi_\pi (\pi_t - \pi^*) + \phi_y (y_t - y_t^*). \quad (1)$$

In the United States, a rule like this is thought applicable to the years of Volcker and Greenspan, 1979 till now. Notice that the variable that is ‘doctored’ is the interest rate, instead of the usually assumed aggregate money supply. This is

thought more in line with the actual practice of monetary policy; also, policy aimed at  $r$  does not depend on a stable demand for money.

It would be outside the scope of this report to fully replicate professor Woodford's lectures. By leaving out the non-essential parts, we will try to replicate the gist of the story, referring the interested reader to Rotemberg and Woodford (1997, 1998) for the details. In the next section, we look at the micro-founded macro model that serves as the workhorse for our analysis. Section 3 shows how this model naturally spawns a criterion by which to judge the outcome of a policy-rule. The feasibility of different rules is discussed in section 4, and in section 5 we look at several counterfactuals run over the period 1979–95. Section 6 concludes.

## 2 A macro model

The aim of this section is to show the construction of a simple macro model, which exhibits the interplay between inflation, aggregate production, and the interest rate. The economy is taken to be a continuum of households, indexed on  $[0, 1]$ . Each household functions like a yeoman farmer, producing one good and trading it with other households. Money is introduced in the model in the Sidrauski-way, by making households like it *per se*. The utility of agent  $j$  is given by

$$U_t^j = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ u(c_t^j) - v(y_t^j) + w \left( \frac{m_t^j}{P_t} \right) \right] \right\} \quad (2)$$

where  $y_t^j$  is agent  $j$ 's production and  $c_t^j$  is a CES aggregate of the different products consumed by the agent:

$$c_t^j = \left[ \int_0^1 c_t^j(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \quad \text{where } \theta > 1. \quad (3)$$

It follows that the market for goods is monopolistically competitive. The functions  $u$  and  $w$  are increasing and convex, and  $v$  is increasing and concave. From the utility function we see that working causes displeasure and the possession of money is valued positively. The price index  $P_t$  combines the prices of goods,  $P_t^j$ , in line with the consumption index in formula (3).

It is assumed that agents have at their disposal the use of risk-free bonds that pay real interest  $r_t$ . The Fisher parity condition implies that nominal interest  $i_t$  satisfies

$$1 + i_t = (1 + r) \frac{P_{t+1}}{P_t}. \quad (4)$$

Thus, agents can shift wealth through time in the form of both money ( $m_t^j$ ) and bonds ( $b_t^j$ ). The budget condition that goes with the problem is

$$b_{t+1}^j + \frac{m_t^j}{P_t} = (1 + r) b_t^j + \frac{m_{t-1}^j}{P_t} + y_t^j - c_t^j, \quad (5)$$

keeping in mind that at time  $t$ , the decision is made how much money to hold now and how many bonds to keep until the next period.

Because the agents are both producers and consumers, maximizing their utility gives the equilibrium of the entire economy. The problem is solved by analyzing three sub-problems:

1. given total expenditure, allocate across the different goods.
2. given the revenues from supplying the goods, find an intertemporal plan for  $\{c_t^j\}$  and  $\{m_t^j\}$ .
3. find the optimal pricing (or supply-) decision each period.

Problem 1 is the standard monopolistic competition problem, and renders

$$c_t^j(z) = c_t^j \left( \frac{P_t(z)}{P_t} \right)^{-\theta} \quad (6)$$

Problem 2 is solved by substituting (5) in (2) and differentiating with respect to  $m_t^j$  and  $b_t^j$ . This gives two efficiency conditions. First, the usual Euler smoothing equation

$$u'(c_t^j) = \beta(1+r_t) E_t \left[ u'(c_{t+1}^j) \right] \quad (7)$$

holds. Secondly, deferring consumption one period and using the freed-up money to invest in bonds should be utility-neutral, so that<sup>1</sup>

$$\frac{w'(M_t^j/P_t)}{u'(c_t^j)} = 1 - \frac{1}{1+i_t} = \frac{i_t}{1+i_t}. \quad (8)$$

## 2.1 Frictionless equilibrium

To solve problem 3, we need to assume a type of price-setting behavior. Assume first that prices are perfectly flexible. Agents solve the problem

$$\max_{P_t^j} \lambda_t^j \left( P_t^j Y_t \left( \frac{P_t^j}{P_t} \right)^{-\theta} \right) - v \left( Y_t \left( \frac{P_t^j}{P_t} \right)^{-\theta} \right)$$

where  $\lambda_t^j$  is the marginal utility of additional money revenue<sup>2</sup> at time  $t$ , and  $Y_t$  is an index for total consumption,  $Y_t = \int_0^1 c_t^j dj$ . The f.o.c., taking into account that  $P_t$  and  $Y_t$  are not affected by changes in  $P_t^j$ , is

$$P_t^j = \frac{\theta}{\theta-1} \frac{v' \left( Y_t \left( \frac{P_t^j}{P_t} \right)^{-\theta} \right)}{\lambda_t^j}.$$

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<sup>1</sup>This condition may be found by taking (2)'s first order condition w.r.t.  $m_t^j$ , and using the Euler equation (7) and the Fisher parity (4).

<sup>2</sup>There holds that  $\lambda_t^j = u'(c_t^j)/P_t$ .

Because in equilibrium, all agents act the same, we can substitute in that  $c_t^j = Y_t$ ,  $m_t^j = M_t$  and  $P_t^j = P_t$  for all  $j$ . This leads to the reduced form macro model with three equations.

The first equation relates money demand and with interest and income: an LM-type equation.

$$\frac{w'(M_t/P_t)}{u'(y_t)} = \frac{i_t}{1+i_t} \Rightarrow M_t/P_t = L(y_t, i_t) \quad \text{LM}$$

The second equation is reminiscent of an IS equation

$$\frac{u'(y_t)}{P_t} = \beta (1 + i_t E) \frac{u'(y_{t+1})}{P_{t+1}} \quad \text{IS}$$

The third equation is the aggregate supply relationship

$$v'(y_t) / u'(y_t) = \frac{\theta-1}{\theta} \quad \text{AS}$$

We see that aggregate supply is constant and independent of monetary policy. This is due to the assumption of price flexibility made earlier. The first two equations of this model are not influenced by that assumption and will still hold later on, when we change it.

The micro-foundations of this model allow us to introduce shocks to different parts of the economy in a straightforward way (rather than tucking them on to the reduced form equations). We introduce three types of shocks:

- $\xi_{1t}$ , shocks to fiscal policy, added to the function  $u$ .
- $\xi_{2t}$ , preference or technology shocks, added to the function  $v$ .
- $\xi_{3t}$ , shocks to the technology of transactions, added to the function  $w$ .

Assuming the shocks and their effects on the endogenous variables are reasonably small, we can rewrite the model in a log-linear approximation

$$\log M_t - \log P_t = \eta_y \ddot{y}_t - \eta_i \ddot{r}_t + \nu_{1t} \quad \text{LM}$$

$$-\sigma \ddot{y}_t = \ddot{r}_t + E_t(-\sigma \ddot{y}_{t+1} - \pi_{t+1}) + \nu_{2t} \quad \text{IS}$$

$$\ddot{y}_t = y_t^S \quad \text{AS}$$

The notation is as follows:  $\ddot{y}_t$  is  $\log(Y_t/Y_t^*)$ , and  $\ddot{r}_t$  is  $\log(1+i_t) - \log(1+i_t^*)$ , where the values with a star are the equilibrium solutions. Further,  $\pi_t$  is the inflation

rate  $\log(P_{t+1}/P_t)$  and  $\sigma$  the curvature of the utility function  $u$  at  $Y_t = Y_t^*$ .<sup>3</sup> The disturbances are assumed to show up as the terms  $\nu_{1t}$  and  $\nu_{2t}$ , and the AS equation shows that output is purely a function of the disturbances.

## 2.2 Frictions in price setting

We now follow Calvo (1983) by assuming the following kind of friction in the setting of prices: rather than letting all producers change their prices every period, a price may be changed with probability  $1 - \alpha$ . With probability  $\alpha$ , the price must stay the same as last period.

When a producer is given the opportunity to change the price, the price that is set is not necessarily the same as the optimal price in the frictionless model. This is because there is a positive probability  $\alpha$  that next time, the price cannot be changed, so that it holds for more than one period. Thus the producer will want to maximize an expression like

$$\max_{P_t^j} E_t \left\{ \sum_{k=0}^{\infty} \alpha^k \beta^k \Pi(P_t^j, P_{t+k}, Y_{t+k}, \lambda_{t+k}, \xi_{2,t+k}) \right\}$$

where  $\Pi$  is the ‘profit’-function.<sup>4</sup> Log-linearizing the above expression around  $Y_t = Y_t^*$  and ploughing through a fair amount of algebra, we obtain a new AS equation, which looks like

$$\begin{aligned} \pi_t &= \kappa (\ddot{y}_t - \ddot{y}_t^S) + \beta E_t \pi_{t+1}, \text{ where} \\ \kappa &= \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \left( \frac{\omega + \sigma}{1 + \omega\sigma} \right) > 0 \end{aligned}$$

and where  $\omega$  is the curvature of  $v$  at  $Y_t = Y_t^*$  and  $\ddot{y}_t^S$  is the solution for  $\ddot{y}$  for the flexible-price case. We can now write a third version of our macro model, subject to price rigidities.

$\log M_t - \log P_t = \eta_y y_t - \eta_i r_t + \nu_{1t}$	LM
$\ddot{y}_t = E_t \ddot{y}_{t+1} - \frac{1}{\sigma} [\ddot{r}_t - E_t \pi_{t+1}] + g_t$	IS
$\pi_t = \kappa (\ddot{y}_t - \ddot{y}_t^S) + \beta E_t \pi_{t+1}$	AS

We have slightly rewritten equation IS, and named the modified disturbance term  $g_t$ . Taking the real side of the economy, IS and AS, we can now show the use of monetary policy rules.

<sup>3</sup>This curvature is given by  $-u''(Y_t^*)/u'(Y_t^*)$ .

<sup>4</sup>Of course these yeoman-farmers do not think of profit in the proper sense, but they do maximize something similar.

### 2.3 A policy rule in the model

The system AS-IS above can be amended with a policy rule for the interest rate,

$$\ddot{r}_t = \phi_\pi \pi_t + \phi_y \ddot{y}_t.$$

Using this rule, we can get rid of the  $\ddot{r}_t$  in IS. Defining  $\mathbf{z}_t = [\pi_t, y_t]'$ ,  $\mathbf{s}_t = [g_t, y_t^s]'$  and substituting in the rule, we have a system that may be represented as

$$\mathbf{z}_t = \mathbf{A}\mathbf{z}_t + \mathbf{B}E\mathbf{z}_{t+1} + \mathbf{C}\mathbf{s}_t. \quad (9)$$

By invoking rational expectations, we can write

$$\begin{aligned} \mathbf{z}_t &= (\mathbf{I} - \mathbf{A})^{-1} \mathbf{B}E\mathbf{z}_{t+1} + (\mathbf{I} - \mathbf{A})^{-1} \mathbf{C}\mathbf{s}_t \\ &= (\mathbf{I} - \mathbf{A})^{-1} \mathbf{C} \sum_{i=0}^{\infty} \left( (\mathbf{I} - \mathbf{A})^{-1} \mathbf{B} \right)^i E_t \mathbf{s}_{t+i} \end{aligned}$$

where the last step holds if  $\|(\mathbf{I} - \mathbf{A})^{-1} \mathbf{B}\| < 1$ . Assuming a VAR process for the shocks, this gives a unique bounded solution for the system. This solution shows how the economy will dynamically respond to a series of shocks  $\{\mathbf{s}_t\}$ .

An important point needs to be made here. Every time the monetary policy rule is substituted in we must have assumed that the rule is known and credible; otherwise it does not work. Would the central bank take the same actions by surprise, the effects would be different. Thus, precommitment by the bank is crucial for the outcome.

## 3 A ranking criterion

With the tools we have developed, we can analyze the workings of the economy under different policy rules. To compare the desirability of the different outcomes, a welfare criterion must be specified. Because of the micro-foundations of our model, this criterion can be found relatively straightforwardly. We could, for instance, look at the maximand of the consumers in formula (2).

Because we will be looking at steady states, an equivalent measure is found by looking at the expression for one period,

$$E \left[ u(c_t; \xi_{1,t}) - \int_0^1 v(y_t^z; \xi_{2,t}) dz + w\left(\frac{m_t}{p_t}; \xi_{3,t}\right) \right].$$

Given that consumers are identical all arguments are aggregate variables. In equilibrium,  $c_t = y_t^z$  for all  $z$ .

A number of assumptions go into the construction of a criterion from this expression. First of all, the  $m_t/p_t$ -term is neglected, focusing attention on the real side of the economy. Furthermore, we will look at only a second order Taylor expansion around a steady state with zero disturbances. A number of other assumptions will follow during the construction.

We start with the terms involving  $u(\cdot)$ . Define  $\hat{y} = \log(y_t/y^*)$ , then it is possible to write the expansion as

$$\begin{aligned} u(y_t; \xi_{1,t}) &= u(y^*, 0) + u_c y^* \left( \hat{y}_t + \frac{1}{2} \hat{y}_t^2 \right) + u_\xi \xi_{1,t} + \frac{1}{2} u_{cc} y^{*2} \hat{y}_t^2 \\ &\quad + u_{c\xi} y^* \hat{y}_t \xi_{1,t} + \frac{1}{2} u_{\xi\xi} \xi_{1,t}^2 + O(\|\xi_{1,t}^3\|) \end{aligned}$$

From this, we can ignore the terms that are independent of policy (t.i.p., terms that do not change as the policy rule is changed), and take expectations to find

$$\begin{aligned} E(u) &= u_c y^* \left[ E(\hat{y}) + \frac{1}{2} (1 - \sigma) E(\hat{y})^2 + \frac{1}{2} (1 - \sigma) \text{var}(\hat{y}) + \sigma \text{cov}(\hat{y}, \bar{c}_t) \right] \\ &\quad + \text{t.i.p.} + O(\|\xi^3\|) \end{aligned}$$

where  $\bar{c}_t \equiv -u_{c\xi} \xi_{1,t} / u_{cc} y^*$ .

A similar computation can be made for the second term, involving  $v(\cdot)$ , which comes out as

$$\begin{aligned} v_y y^* [E(\hat{y}) + \frac{1}{2} (1 + \omega) E(\hat{y})^2 + \frac{1}{2} (1 + \omega) \text{var}(\hat{y}) - \omega \text{cov}(\hat{y}, \bar{y}_t)] \\ + \frac{1}{2} (\theta^{-1} + \omega) E\{\text{var}_z(\log y_t^z)\} + \text{t.i.p.} + O(\|\xi^3\|) \end{aligned}$$

where  $\bar{y}_t = -v_y \xi_{2,t} / v_{yy} y^*$ .

Two further assumptions now facilitate the summation of these two terms. First of all, assume that  $u_c = v_y$ . This means that the steady-state level of output is always efficient, in spite of the shocks. This may be caused by the use of other instruments. Secondly, assume that other policy instruments are adjusted such that  $E(\hat{y}) = 0$  regardless of monetary policy. This greatly simplifies the criterion, which may now be written as

$$\begin{aligned} \mathcal{W} &= -\frac{u_c y^*}{2} \left[ \underbrace{(\sigma + \omega) \text{var}(\hat{y} - \hat{y}^s)}_{\text{Variability of the output gap}} + (\theta^{-1} + \omega) \underbrace{E(\text{var}_z(\log y_t^z))}_{\text{Dispersion of output levels}} \right] \\ &\quad + \text{t.i.p.} + O(\|\xi^3\|) \end{aligned}$$

The term that quantifies the impact of the dispersion of output levels is only invoked when there is some sort of price rigidity causing the dispersion. Until now we have not made any assumptions about the way in which prices are set. We will use the Calvo-mechanism we have used before to allow for some rigidity. This means that each period, a fraction  $\alpha$  of the producers cannot change their price.

The appearance of  $\mathcal{W}$  changes somewhat after the Calvo-rule is used. It now reads

$$\mathcal{W} = -\frac{u_c y^*}{2} \frac{\alpha}{(1 - \alpha)^2} (\sigma^{-1} + \omega) [\mathcal{L} + \pi^{*2}] + \text{t.i.p.} + O(\|\xi^3\|)$$

Here,  $\mathcal{L}$  may be viewed as the actual stabilization loss, and  $\pi^{*2}$  as the average rate of loss. It turns out that

$$\mathcal{L} = \frac{(1 - \alpha)^2}{\alpha} \left( \frac{\sigma + \omega}{\theta^{-1} + \omega} \right) \text{var}(\hat{y} - \hat{y}^s) + \text{var}(\pi).$$

The result that the loss function ultimately depends on a weighted average of the variation in output and the variation in inflation is not unfamiliar in macroeconomics. It has often been assumed without a micro-foundation that such a criterion could be relevant. The advantage of this form is that the weights of the two variations are given by the model, rather than picked by the economist.

Note that we started out by omitting losses due to monetary fluctuations, looking only at production and consumption. However, the criterion that we found shows that there still is an aversion to inflation (both in level and in variation).

## 4 Feasibility of policy rules

Now that we have a criterion by which we can rank the different rules, we might want to look for the rule that minimizes it. This would be the rule that caused zero inflation in every period. However, we must take into account that only a limited amount of information about the economy is known, so that in practice it may not be possible to formulate a rule that produces  $\pi = 0$ .

We again post a version of our macro model, leaving out the LM equation<sup>5</sup> but adding a policy rule.

$\pi_t = \kappa \tilde{y}_t + \beta E_t \pi_{t+1}$	AS
$\tilde{y}_t = -\frac{1}{\sigma} [r_t - E_t \pi_{t+1} - r_t^n] + E_t \tilde{y}_{t+1}$	IS
$r_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + q_t$	PR

where  $\tilde{y}_t = y_t - y_t^s$  is deviation in output and  $r_t^n = \sigma (g_t + E_t y_{t+1}^s - y_t^s)$  is the combined effect of all disturbances. The rule PR allows for an exogenous shock  $q_t$  by the policy makers.

<sup>5</sup>The LM equation determines the demand of money but does not influence the real economy in this model.

We can now explicitly write down a system such as (9). There holds that

$$\begin{aligned} \begin{pmatrix} \pi_t \\ \tilde{y}_t \end{pmatrix} &= \tilde{\mathbf{B}} \begin{pmatrix} E_t \pi_{t+1} \\ E_t \tilde{y}_{t+1} \end{pmatrix} + \tilde{\mathbf{C}} \begin{pmatrix} q_t \\ r_t^n \end{pmatrix} \\ &= \frac{1}{\Delta} \begin{pmatrix} \frac{\beta\sigma + \kappa + \beta\phi_y}{1 - \beta\phi_\pi} & \kappa \\ \sigma & 1 \end{pmatrix} \begin{pmatrix} E_t \pi_{t+1} \\ E_t \tilde{y}_{t+1} \end{pmatrix} - \frac{1}{\sigma} \begin{pmatrix} \kappa \\ 1 \end{pmatrix} (q_t - r_t^n) \\ \Delta &= \frac{\sigma + \kappa\phi_\pi + \phi_y}{\sigma} \end{aligned}$$

From here we can answer a number of important questions. One question might be whether the economy has a determinate rational expectations solution. The answer lies in the properties of matrix  $\tilde{\mathbf{B}}$ : if both eigenvalues lie inside the unit circle such an equilibrium exists. It can be shown that this condition is satisfied if  $\phi_\pi > 1$  and  $\phi_y \geq 0$ ; this is not a necessary condition, however.

Another question, given that a solution exists, might be what the effect of the original shocks  $\xi_1$ ,  $\xi_2$  and  $\xi_3$  would be on output and inflation. The answer to that question can be found by examining the effect of the  $\xi$ 's on  $r_t^n$  and looking at  $\tilde{\mathbf{B}}$  and  $\tilde{\mathbf{C}}$ .

The effects of exogenous shock  $q_t$  can also be derived from this system. It appears that a policy strategy that sets  $q_t = r_t^n$  for all  $t$  would result in the minimum possible loss criterion, for all values of  $\phi_\pi$  and  $\phi_y$  that generate a determinate solution. The practical feasibility of such a policy is questionable, however.

## 5 Quantification and counterfactuals

Now that the model is completed, professor Woodford showed how we can analyze real-world monetary policy in terms of policy rules, and try how alternative rules might have fared under the shocks that actually occurred. The methodology runs as follows:

1. Estimate an unrestricted VAR model of the interest rate, the inflation rate and output.
2. Choose the parameters of the structural model so that the predicted response to monetary policy agrees with the VAR estimate.
3. Using the quantitative specification of the structural model, and the VAR to model the expectations, identify the shocks that actually occurred by the residuals of the equations when fit to the data.
4. The parametrized model and the shocks can now be used to simulate the consequences of counterfactual monetary policy rules for the evolution of  $\{r_t, \pi_t, y_t\}$ .
5. Using the criterion derived in section 3, evaluate the welfare consequences of alternative policy regimes.

The period over which the analysis is conducted is 1979–95. The model that is used is a little different from the one described above, incorporating a two-period decision lag for goods purchases. This adaption was made to accommodate the data. Also, there are decision lags in pricing by suppliers, which differ across goods. Without these adaptations, shocks would affect the real economy almost immediately; this is contradicted by the data.

We already noticed that a rule that causes zero inflation would be infeasible, because there is not enough information to ‘fuel’ such a rule. We could solve for the best value of  $\mathcal{W}$  given that the rule can only depend on known variables. However, it turns out that such a rule will cause negative interest rates at certain points in time: a practical impossibility.

To avoid this problem, we can impose a condition that adds a penalty for interest rate variability to the loss-criterion. The optimal rule that comes out this problem is indeed feasible, but not very practical as it is very complicated (imagine how it uses *all* available information to come up with the best interest rate).

The rules that validate real interest are those that are simple in structure, yet imply a value for the loss criterion that is close to optimal. In Rotemberg and Woodford (1998), three such rules are discussed:

1. A generalized Taylor rule:  $r_t = a\pi_t + by_t + cr_{t-1}$ .
2. Dependence on lagged data:  $r_t = a\pi_{t-1} + by_{t-1} + cr_{t-1}$ . This is the above rule in the case where data becomes available with a lag.
3. Price level targeting:  $r_t = aP_t + by_t + cr_{t-1}$ . This rule can also be applied in a ‘lagged’ version.

In each case the question is what the optimal parameters  $a$ ,  $b$ , and  $c$  are, whether the rule generates determinate paths for the state variables, and how much the loss criterion is minimized. We look only at the rules in case 1, the others may be found in the paper.

For rules in category 1 with  $c$  set to zero, the optimal values are  $a = 2.88$  and  $b = 0.02$ . This is different, but not shockingly so, from Taylor’s (1993) values. Rotemberg and Woodford (1998) show how the loss criterion behaves for different  $a$ ’s and  $b$ ’s in a diagram. The value of the loss criterion gets very close to that attained in the optimal case discussed above. Rules in category 1 with  $c$  unequal to zero include a feedback mechanism in  $r_t$ . Surprisingly, diagrams show that the rule generates stable paths for values of  $c$  as high as 10! This is due to the fact that the rule is a *promise* to raise interest rates, which is not necessarily carried out. The optimal value is close to the previous one:  $a = 1.22$ ,  $b = 0.06$  and  $c = 1.28$ .

For the above optimal rules, the counterfactual loss criterion is lower than it was in the real case. This suggests that something can be gained studying these optimal rules.

## 6 Conclusions

At the end of the lectures, a number of important conclusions could be drawn. The most striking result probably is that a simple (backward-looking) Taylor-type rule can achieve outcomes that are nearly as good as those achievable by any policy. This suggests that those rules can be a valuable yet inexpensive tool for central banks.

It is often suggested that a central bank's policy should be forward-looking, that the bank should carry out 'preemptive strikes' before a problem actually occurs. This model shows that such is not necessary, as long as the economic agents themselves are forward looking. The promise of the central bank to raise interest rates in certain cases, laid down in a Taylor rule, is enough to stop those cases from happening. The credibility of such a promise is, of course, essential.

Because data often takes time to reach the economist, it is reassuring that the performance of the rules is but weakly affected by the use of lagged data.

Rules that specify short-term interest rates should incorporate the lagged endogenous variable as a regressor, with a coefficient larger than one. Because of rational expectations, this will not result in instability of the economy. There is little gain from making the rule dependent on realized values of aggregate production.

Of course all these conclusions stand or fall with the faith that one puts in the simple macro model that they come from. However, the ability of the model to mimic real data is encouraging, as is the fact that Taylor rules can very well explain past monetary policy. Thus, applying them in the future should not be a step in the dark.

The lectures that professor Woodford gave in Wageningen were by no means 'an easy ride,' but with this impressive list of conclusions well worth the effort. Future data will show whether this work has had impact on the workings of, say, the European central bank.

## 7 References

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